Math-GAMES Compendium

Games and Mathematics in Education for Adults

Funded by the Erasmus+ Programme of the European Union

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Math-GAMES Compendium

GAMES AND MATHEMATICS IN EDUCATION FOR ADULTS
COMPENDIUMS, GUIDELINES AND COURSES
FOR NUMERACY LEARNING METHODS BASED ON GAMES

ENGLISH

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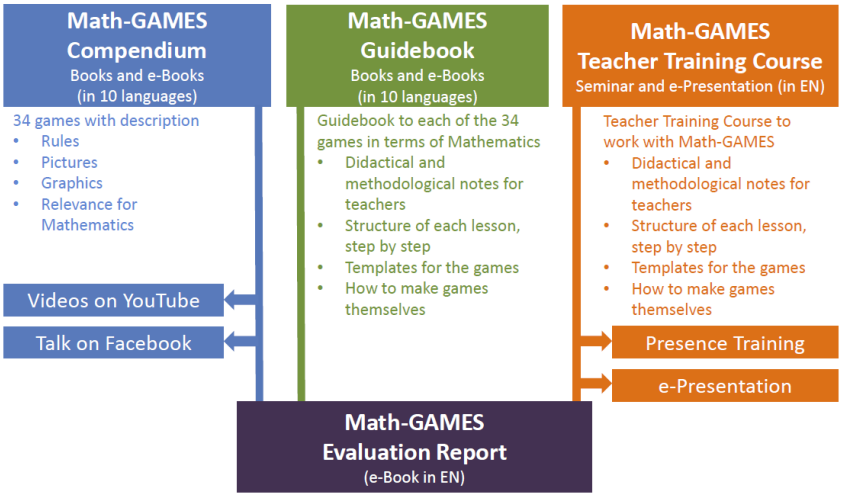
Structure of the European Erasmus+ Project Math-GAMES

How games can help numeracy
- learning to count and calculate,
- learning basics in Mathematics, Statistics and Geometry



The complete output of the project Math GAMES consists of the here present **Compendium** and a **Guidebook**, a **Teacher Training Course and Seminar** and an **Evaluation Report**, mostly translated into nine European languages. You can download all from the website www.math-games.eu

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Preliminary Remarks

Contribution for the preparation of this Compendium

The Guidebook is the outcome of the collaborative work of all the Partners for the development of the European Erasmus+ Math-GAMES Project, namely the following:

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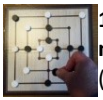
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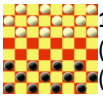
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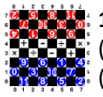
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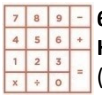
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7.1 Magic Square
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7.2 Four Seasons
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10.1 Okey
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10.2 Hopscotch
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10.3 Sudoku
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Link to the special Math-GAMES YouTube Channel:
https://www.youtube.com/channel/UCvuYRVDPN_WRNO5SwQiRre4g

PREAMBLE

Using games to develop numeracy

Games can help learners to practice matching, counting and computational skills such as doubling, addition, subtraction and tables. Some games combine these skills with strategy, and this can help learners to develop problem-solving skills. Children's boards' games or dominoes can be used in family numeracy sessions. Adult games include bingo, dominoes, card games, strategy games such as backgammon, and traditional African games such as Oware and Ayo, which are now available commercially.

Statement from the "Adult Numeracy Core Curriculum", London, 2001

More than 13% of all people in Europe cannot read, write or count. Therefore it is the declared goal of the European Union to remedy this situation and to reduce the number of weakly trained people. In this environment, the project Math-GAMES is settled. Its title says it all: "***Math Games - Games and Mathematics in Education for Adults - Compendium, Guidelines and Courses for Numeracy Learning Methods Based on Games (Mathematical Literacy)".*** In the project books and handouts will be created, such as the present compendium together with the guidebook, which should give an answer in nine languages to the following questions:

1. How can we reduce the number of under skilled adults to promote social integration and participation into our society?
2. How can we increase incentives for adult training by using games?
3. How can we offer tailored learning opportunities to individual learners by using games?
4. How can we provide information on access to the services of adult learning?
5. How can we save traditional and famous games in different countries from a loss?

The authors of this compendium hope now that the users will have much joy in playing our games, because joy helps you to learn. In addition the authors hope to make a contribution that more people can apply basic mathematical content through this compendium.

Roland Schneidt on behalf of the writing staff in June 2016

PROLOGUE

The present “Compendium of Math-Games for Adults” is first of all a fine and inspiring book of games. While I read it I took a standard collection of games out of the cupboard and realized that by now my son can play Nine Men's Morris and Checkers with me. I discovered Connect Four at playgrounds and after many years played Combination Nine again with my friends.

These well-known games firstly train our strategic abilities. Furthermore, the authors of this compendium detected a new dimension: They show us which *mathematical* contents can be made accessible through these games. In this context it is noticeable that the games that have not been especially developed for math-lessons do not emphasise on pure math. In mathematical terms they hardly stretch beyond counting, basic geometrical terms and the recognition of dice-figures. If one wants to apply math to these games one has to be willing to ask certain questions and make some suggestions. Then you suddenly realize that Ludo and its variations become faster and more interesting (and more rewarding for a structured view of quantities) if you move your man not in single steps but in larger units. If you actually *talk* about Tangram, geometrical terms suddenly come to life and dynamice your own ability to copy the shapes. By playing Chess calculating only becomes important when one attaches numeric values to the various figures.

It is a comforting thought that traditional games allow the player to apply math but also leave the option to stay away from math: In further education there are not only students that are keen to solve all arithmetical and geometrical tasks. There are also the ones that are deeply frustrated by them or are afraid of all mathematical issues. The games of the compendium offer those people the possibility to stay away from mathematical issues surrounding the games - or maybe get a bit closer in a later stage. In addition to the traditional and rather strategy-orientated games the compendium is also a collection of games that apparently were specifically developed to promote mathematical competence amongst the players. The compendium does pioneering work in making such games internationally known. The game DAMATH – a sort of extension of Checkers, becomes a game that promotes calculating - is for example very

popular in the Philippines but due to the compendium it will also become better known in Europe. The basis of “Calculator Hopscotch” is the idea to move forward in order to have new mathematical tasks. One's first thought might be: “Will my students let themselves get engaged in such a thing?” But working with adults allows you to try out a lot of things: On one hand I am always surprised by the various ways games are taken up. Some games that leave me totally cold are enthusiastically welcomed by other adults and games that I absolutely love bore others (and raise joyful expectation in some others). Adults also know that it is *not only* the fun-factor that makes them play. They are able to reflect the mathematical gains they expect to have from a game and still have fun playing it. They can even gain more fun playing a game that they already know when they realise that in addition to the fun-factor the game provides a mathematical dimension that is new to them.

In addition the compendium promotes a third type of game, namely active games such as The Crow´s Feet, Skipping Rope, Hora, Seven Steps or Hopscotch. They do not only provide us with ideas how to “loosen up” our lessons. They mainly promote mathematical thinking in a new way: The students develop a sense of structure. This approach corresponds with the conceptual design of the German “Curricular Framework for Calculating” for adult education at adult education centres (see http://grundbildung.de/material/rechnen.html).

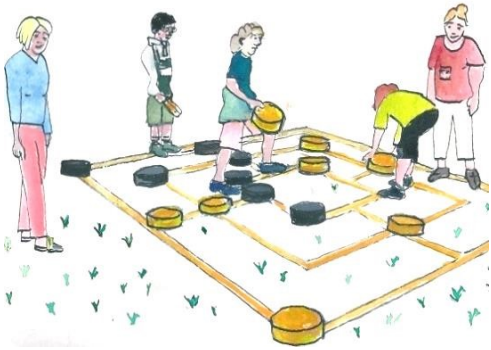
The basic idea was that adult students have not been successful when taught in the traditional way during traditional math lessons. Each individual's concept of numbers and calculating – and therefore each individual's structural thinking – has to be the starting point for learning maths. The games of the compendium offer numerous starting points to take up this individual structural thinking and to promote it further.

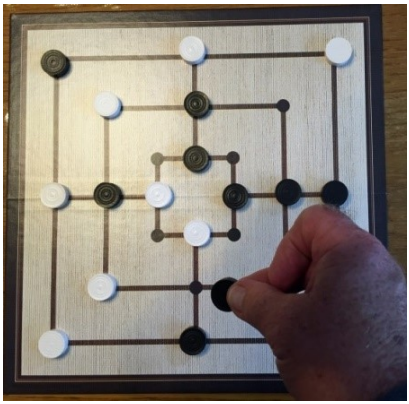
I wish this compendium a widespread distribution and the readers a lot of pleasure in discovering, rediscovering and rethinking these games.

Wolfram Meyerhöfer

Prof. Dr. Wolfram Meyerhöfer is professor for math-didactics at the University of Paderborn and co-editor of the curricular framework for calculating of the association of German adult education centres DVV.

1.1 Nine Men's Morris (Mill)





Nine men's morris or Mill is a very old game, to be played outside or on a table board.

Overview

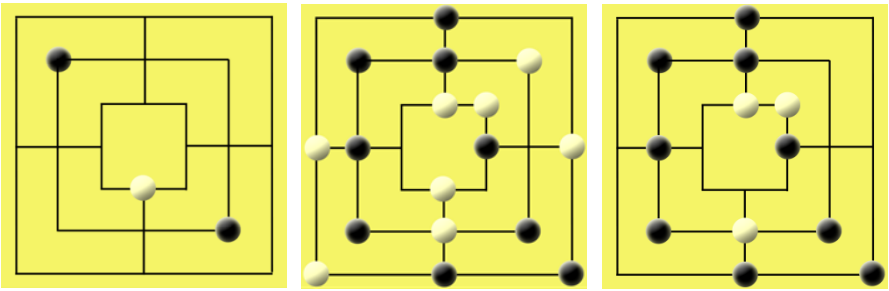
- **Genre:** strategy board game
- **Players:** 2, each has 9 pieces (men)
- **Age range:** older than 5
- **Setup Time:** less than 1 minute
- **Playing Time:** 3 minutes to 1 hour
- **Random Chance:** none, no dice needed

Game rules

General

Mill is a board game for 2 players. There is a board on the table and 2 players sitting vis-à-vis. Each player has 9 tokens or pieces. The tokens are in different colours, usually black and white. The board consists of a grid with three squares and four lines. Squares and lines together make twenty-four intersections or

points. The intention of the players is to form “mills” that means, three of their own tokens in the same colour lined horizontally or vertically. If you get a “mill”, it allows the player to remove an opponent's token from the game immediately, and it is obligatory.



Phase 1: Placing tokens *Phase 2: Moving tokens* *Phase 3: Jumping tokens*

A player wins by reducing the opponent to two tokens, where he can no longer form mills and thus is unable to win, or by putting him in a position where he is unable to move within the rules.

The game proceeds in three phases:

- 1st phase is placing tokens on vacant points
- 2nd phase is moving tokens to adjacent points
- 3rd phase means jumping tokens from any point to any vacant point.

1st Phase: placing tokens

The game begins with an empty board. The players determine who plays first, and then take turns placing their tokens one per play on empty points. If a player is able to place three of his tokens in a straight line, vertically or horizontally, he has formed a mill and has to remove one of his opponent's tokens from the board and the game. Any token can be chosen for removal, but if possible, not a token in an opponent's mill.

2nd Phase: moving tokens

Players continue to alternate moves, this time moving a token to an adjacent

point. A token may not jump another token. Players continue to try to form mills and remove their opponent's tokens in the same manner as in phase one. A player may 'break' a mill by moving one of his tokens out of an existing mill, then moving the token back to form the same mill a second time or more. Each time the player removes one of his opponent's tokens. The act of removing an opponent's token is sometimes called "pounding" the opponent.

3rd Phase: jumping tokens

When a player is reduced to three tokens, there is no longer a limitation on that player to move to only adjacent points: The player's tokens may "fly", "hop", or "jump" from any point to any vacant point, e.g. into a "waiting mill".

Strategy

There is no overall strategy, but you always have to concentrate.

At the beginning of the game it is more important to place tokens in versatile locations rather than to try to form mills immediately and make the mistake of concentrating one's tokens in one area of the board.

An ideal position, which typically results in a win, is to be able to shuttle one token back and forth between two mills, removing a token every turn.

More information and a game example

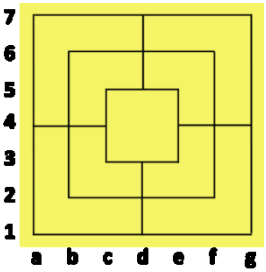
YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Nine men's morris Game instruction: https://youtu.be/eUz0LBza0pk

What Mathematical Content can be Learned?

Numeracy

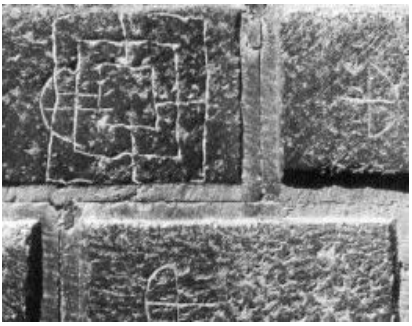
- Count reliably up to 9 items
- Read and write numbers up to 9
- Order and compare numbers up to 10
- Add single-digit numbers with totals to 10
- Subtract single-digit numbers from numbers up to 10
- Draw a number line and sort numbers



- Understand the coordinate system

GEOMETRY

- Line, vertical line
- Right angle
- Recognize and name two dimensional shapes (square, rectangle, circle, circle parts (sector)
- Amount of square and rectangle



Roman Tower in Regensburg,

HISTORY

The earliest known board for the game includes diagonal lines and was cut into the roofing slabs of the temple at Kurna in Egypt c. 1400 BCE, although some people doubt this.

One of the earliest references to the game may be in Ovid's Ars Amatoria. In book III (c. 8 CE), after discussing "Latrones", a popular board game, Ovid wrote: "There is another game divided into as many parts as there are months in the year. A table has three pieces on either side; the winner must get all the pieces in a straight line."

"Picture stone" from Ernstkirchen, Germany, estimated around 800 A.D.

The game was probably well known by the Romans, as there are many boards on Roman buildings, even though dating them is impossible because the buildings have been easily accessible since they were built. It is possible that the Romans were introduced to the game via trade routes, but this cannot be proved.

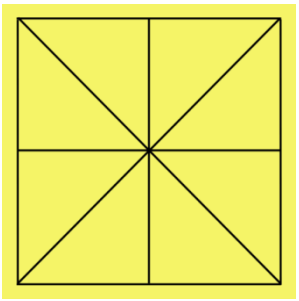
The game peaked in popularity in medieval England. Boards have been found

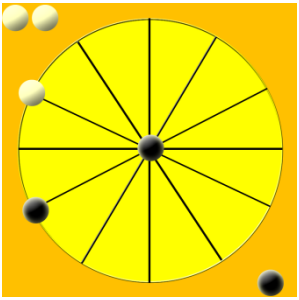
carved into the cloister seats at the English cathedrals e.g. at Canterbury and Westminster Abbey. These boards used holes, not lines, to represent the nine spaces on the board - hence the name "nine holes" - and forming a diagonal row did not win the game.

Giant outdoor boards were sometimes cut into village greens. In Shakespeare's 16th century work “A Midsummer Night's Dream”, Titania refers to such a board: "The nine men's morris is filled up with mud" (A Midsummer Night's Dream, Act II, Scene I).

Some authors say the game's origin is uncertain. It has been speculated that its name may be related to Morris dances.

In some European countries, the design of the board was given special significance as a symbol of protection from evil, and to the ancient Celts, the Morris Square was sacred: at the centre lay the holy Mill or Cauldron, a symbol of regeneration; and emanating out from it, the four cardinal directions, the four elements and the four winds.





Mola Rotunda - the old Romans liked to play this game

Synonyms

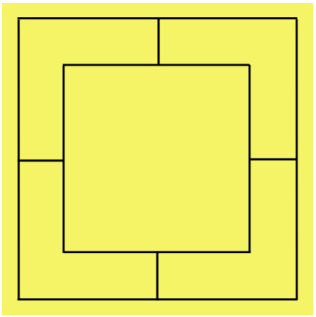
The game is also known as Nine Man Morris, Mill, Mills, The Mill Game, Merels, Merrills, Merelles, Marelles, Morelles and Ninepenny Marl in English.

Variants

Three Men's Morris

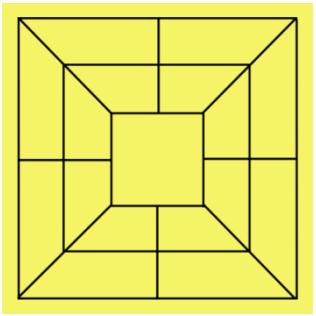
Three Men's Morris, also called "Nine Holes", is played on the points of a grid of 2×2 squares, or in the squares of a grid of 3×3 squares as in tic-tac-toe. The game is for two players; each player has three men. The players put one man on the board in each of their first three plays, winning if a mill is formed (as in tic-tac-toe). The

Old Roman version of this game has more lines.



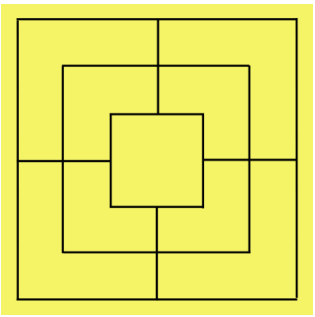
After that, each player moves one of his men, according to one of the following versions of the rules:

- To any empty position.
- To any adjacent empty position (i.e. from an edge position to the centre, or from the centre to an edge position, or from an edge position to an adjacent edge).
- A player wins by forming a mill.



Six Men's Morris

Six Men's Morris gives each player six pieces and is played without the outer square found on the board of Nine Men's Morris. Jumping is not permitted. The game was popular in Italy, France and England during the Middle Ages but was obsolete by 1600. This board is also used for Five Men's Morris. Seven Men's Morris uses this board with a cross in the centre.



Twelve Men's Morris

Twelve Men's Morris adds four diagonal lines to the board and gives each player twelve pieces. This means the board can be filled in the placement stage; if this happens the game is a draw. This variation on the game is popular amongst rural youth in South Africa where it is known as “Morabaraba” and is now recognized as a sport in that country.

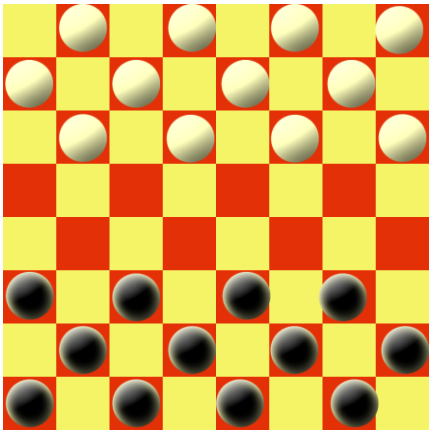
References and Links

Wikipedia: https://en.wikipedia.org/wiki/Nine_Men%27s_Morris

Board Game Geek: http://www.boardgamegeek.com/

Brettspielnetz: http://www.brettspielnetz.de/spielregeln/muehle.php

1.2 Checkers (Draughts)



Checkers or Draughts is an old board game, which exists in different variations. Starting position on an 8 by 8 board and removing a piece by jumping over it.

Overview

- **Genre:** strategy board game
- **Players:** 2, each has 12 pieces (tokens)
- **Age range:** older than 5
- **Setup Time:** less than 1 minute
- **Playing Time:** 3 minutes to 1 hour
- **Random Chance:** none, no dice needed

Game rules

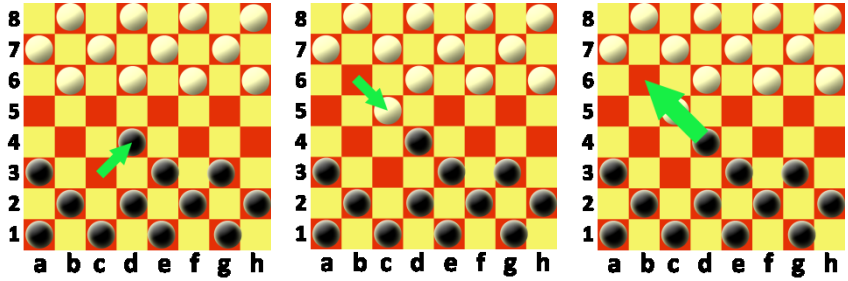
Checkers is an abstract strategy game where players move disc-shaped pieces across an 8 by 8 cross-hatched ("checker") board.

One player has 12 dark pieces, the other has the 12 light ones in the starting position (see picture above).

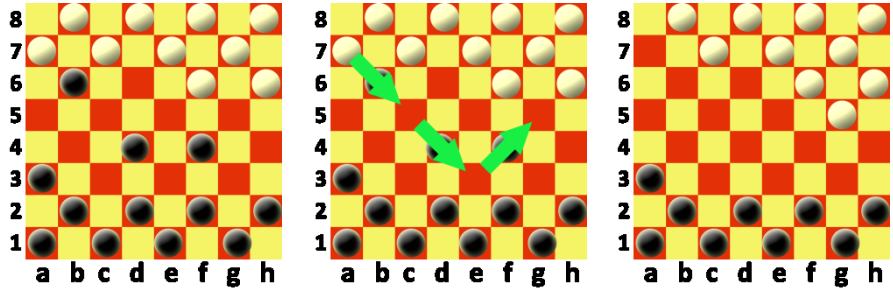
Pieces only move diagonally forward and only one space at a time. Only the dark squares of the board are used.

Jump and Remove

If a player can move one of his pieces so that it jumps over an adjacent piece of his opponent and into an empty space, that player captures the opponent's piece and removes it from the board. Jumping moves must be taken when possible. This capturing is mandatory, if the player will not do this or forgets to do this, the other player can remove the opponent's piece from the board.



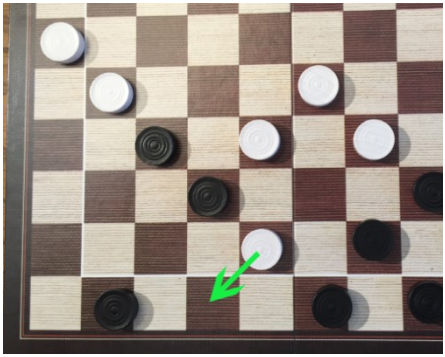
The phases of jumping and removing tokens: black is moving c3-d4; white is moving b6-c5; black is jumping from d4 over c5 to b6 and removes c5 from the board.



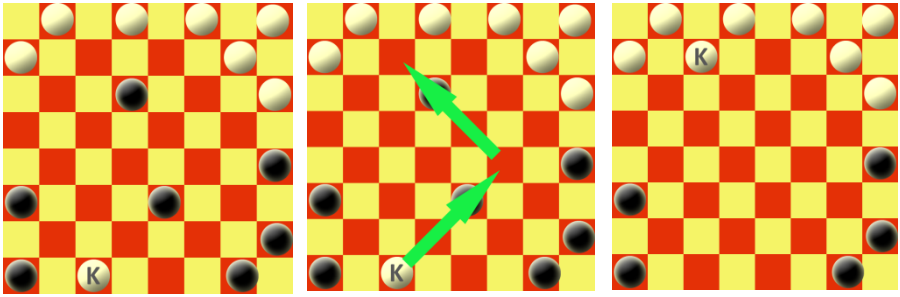
It is possible to jump during one move over more than one opponent's pieces, if there is always one empty square between. The situation left shows empty squares c5 and e3 between black pieces. Middle: white can jump now with a7 over b6, d4 and f4 to g5 and removes three black tokens (right picture).

Men and Kings

The starting situation shows 12 black and 12 white men (pieces, tokens). The men are only allowed to move forward. If one man reaches the last line opposite his side, he is in the so called king's row, and changes into a king and is marked by placing an additional piece on top of the first man. This king now has additional powers including the ability to move backwards, jump over more than one empty square and capture forwards and backwards. As with normal men, a king may make successive jumps in a single turn provided that each jump captures one or more opponent man or king.



If a man reaches the king's row, he is turning from a man into a king by placing an additional piece on his top.



Three steps of the king's move: white king on c1, can jump over e3 and c6, end of this move is white king on c7, e3 and e6 are removed.

Strategy

Players create a strategy when they offer up jumps in exchange for setting up the board, so that they can jump over even more pieces on their turn. The player without pieces remaining, or who cannot move, loses the game.

Another strategy is, to get as many kings as possible, because these kings have more power to move and jump.

More information and a game example

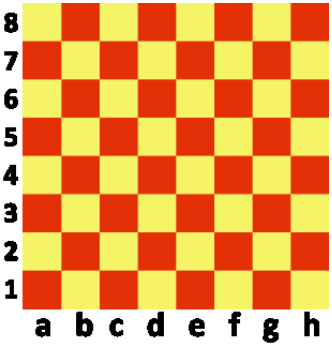
YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Checkers Game instruction: https://youtu.be/Rmn_MkZZ7iU

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 12 items
- Read and write and compare numbers up to 8
- Add single-digit numbers with totals to 8
- Subtract single-digit numbers from numbers up to 8
- Multiplication from 2 to 8
- Draw a number line and sort numbers
- Understand the coordinate system



Geometry

- Recognize and name two dimensional shapes (square, rectangle, regular hexagon)
- Measure areas by counting squares or using grids
- Understand and compare right angles

History

A similar game has been played for thousands of years. A board resembling a draughts board was found in Ur dating from 3000 BC. In the British Museum are specimens of ancient Egyptian checkerboards, found with their pieces in burial chambers. The Roman Plato mentioned a game, πεττεία or petteia, as being of Egyptian origin and Homer also mentions it. The Romans played a derivation of petteia called latrunculi or the game of the Little Soldiers.

An Arabic game called *Quirkat* or *al-qirq*, with similar play to modern draughts, was played on a 5×5 board. It is mentioned in the 10th century.

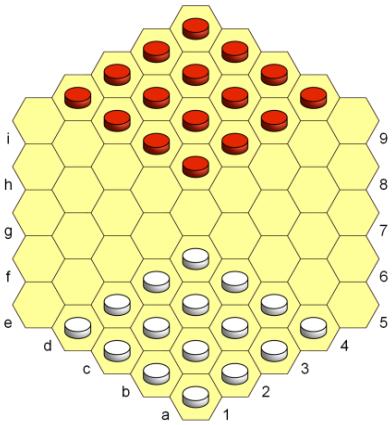
The rule of crowning was used by the 13th century. The pieces became known as "dames" when that name was also adopted for the chess queen. The rule forcing players to take whenever possible was introduced in France in around 1535, at which point the game became known as *Jeu forcé*, identical to modern English draughts. The game without forced capture became known as *Le jeu plaisant de dames*, the precursor of international draughts.

Synonyms

In most non-English languages draughts is called dame, dames or a similar term that refers to ladies. The pieces are usually called men, stones or a similar term; men promoted to kings are called dames or ladies. In these languages, the queen in chess or in card games is usually called by the same term as the kings in draughts.

VARIANTS

HEXDAME



Hexdame is a literal adaptation of international draught to a hexagonal game board. Here are the rules:

- White moves first, then moves alternate; a player may not skip a turn.
- Men move forwards (not backwards). If a man reaches the last row of the board at the conclusion of a move, it promotes to king; the promoted man is crowned for identification as king by placing a second piece of the same colour on top.
- Jumping and capturing is mandatory, including making multiple jumps if available. If there are jumping options, the player must always select the option that captures the maximum number of the opponent's pieces (men or kings); if two options capture the same number, the player may choose.
- A multi-jump can consist of single jumps having a combination of different directions.
- Pieces jumped in a multi-jump are removed from the board only after the jumping move is complete.
- A king can make a flying king move (move any number of contiguous empty cells) or capture (capture a piece any number of empty cells' distance away).
- When a king jumps, it may land on any empty cell beyond the jumped piece, unless there is a requirement to land on a particular cell, in order to continue jumping according to the rule mandating, that all jumps capture maximum pieces.
- During a multi-jump, a piece may not be jumped more than once. Empty cells, however, may be visited or jumped any number of times.

- A player whose pieces all become captured, or is unable to make a legal move, loses the game.
- If the players agree, or if the same position repeats three times with the same player having the move, the game is a draw.

Differences to Draught

- Though Hexdame precisely mirrors the rules of international draughts, the hex board geometry introduces important differences in play dynamic:
- The game has greater complexity, since men have three directions for moving forward, instead of two and kings have six directions of movement, instead of four.
- A single man cannot thwart an opponent's man from advancing through the threat of being jumped, as it can in international draughts, since the opponent will always have another option for moving forward.
- Each side has nine possible promotion cells on the hex board versus five on the 10×10 orthogonal board.
- Draws are fewer, since three kings defeat one king in Hexdame. Three kings versus one in international draughts is a declaration of draw; four are usually needed.

Losing draughts

Losing Droughts is the opposite version of checkers. The winner is the first player, who has no legal move: that is, all of whose pieces are lost or blocked.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/Draughts

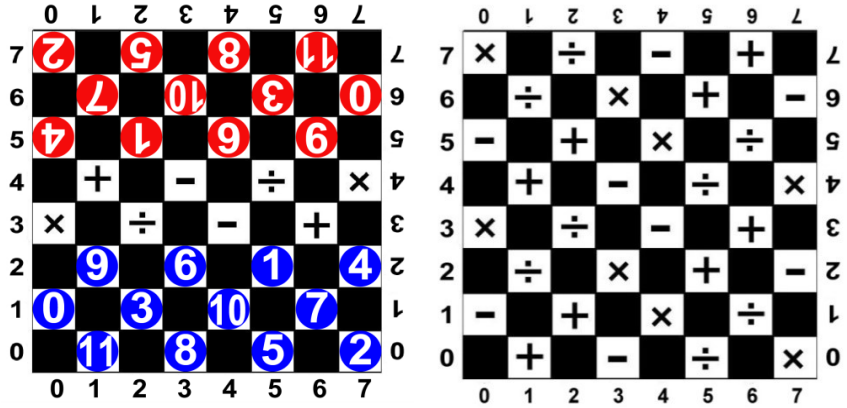
Board Game Geek: http://www.boardgamegeek.com/

Brettspielnetz: http://www.brettspielnetz.de/spielregeln/dame.php

Draughts for Computers: http://www.spielen.de/denkspiele/dame/

http://www.memory-improvement-tips.com/free-internet-checkers.html

1.3 DAMATH



Damath is a variant of Checkers or Draught: Each white field is marked with one of the four elementary types of calculation + (plus), - (minus), × (times) and ÷ (divided by). Each piece has a number on it. The game is very popular in the Philippines and by playing DAMATH people can learn calculating very easily.

OVERVIEW

- **Genre:** strategy board game with the aim to learn counting
- **Players:** 2, each has 12 pieces (tokens)
- **Age range:** older than 5
- **Setup Time:** less than 1 minute
- **Playing Time:** 40 minutes exactly, each of the 20 moves for each player is a one-minute-move.
- **Random Chance:** none, no dice needed

GAME RULES

Damath is a variant of Checkers utilizing math principles and numbered chips popular in the Philippines. It was invented by Jesus L. Huenda, a teacher, who had encountered problems in teaching math using traditional teaching methods.

The name comes from Dame and Mathematics: DaMath. It is an abstract strategy game to learn counting, where players move disc-shaped pieces across an 8 by 8 cross-hatched ("checker") board. Sometimes the boards are 10 by 10. Each piece has a number on it and the white fields of the board are marked with the four elementary types of calculation + (plus), - (minus), × (times) and ÷ (divided by). The basic knowledge for playing the game Damath is the ability to play checkers properly.

Start

One player has 12 red pieces, the other has the 12 blue ones in the starting position (see picture).

Move

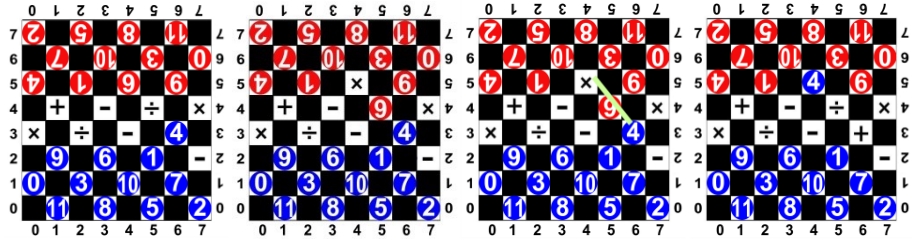
Pieces only move diagonally forward and only one space at a time. Only the white squares of the board are used.

Jump, Remove Pieces and Calculate

If a player can move one of his pieces so that it jumps over an adjacent piece of his opponent and into an empty space, that player captures the opponent's piece and removes it from the board. Jumping moves must be taken when possible. This capturing is mandatory, if the player will not do this or forgets to do this, the other player can remove the opponent's piece from the board. So far it is like the rules of international checkers. But here are the different rules:

- It is a 40-minute gameplay.
- Every piece has a corresponding number from 0 to 11, start position see picture!

- One minute is allotted to each player to make a move.
- Every move is recorded on a piece of paper besides the board.
- When capturing the opponent's piece, the score is obtained by calculating the number value of player's piece which captures the opponent's piece and the captured piece. The mathematical operation used depends where the player's piece lands after the capture.
- The game is over when 20-minute gameplay is lapsed, the player resigns, the player has no more pieces, repetitive moves, or the last opponent's piece is cornered.
- Final score is determined by adding all the value of the remaining piece against the scores obtained from capturing opponent's piece.
- The one who has highest score is the winner.



This could be the first move in Damath:

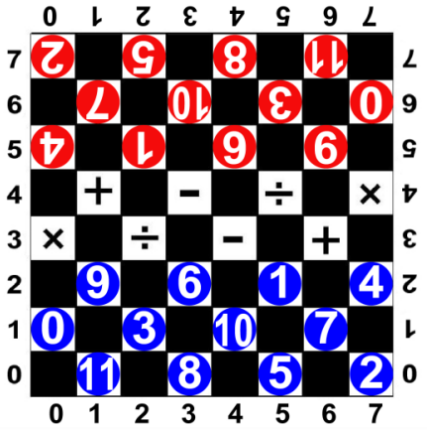
- Blue (7,2) begins and moves to (6,3)
- Red (4,5) moves to (5,4)
- Blue (6,3) jumps over red (5,4) and removes it
- At the same time blue gets 4×6=24 points, because blue moves with blue piece value 4 to field (4,5), which is multiplication ×, over red piece value 6, which is removed.
- Blue records all moves on a paper: 4×6=24, so blue gets 24 points …

Men and Kings

The starting situation shows 12 blue and 12 red numbered men (pieces, tokens). The men are only allowed to move forward. If one man reaches the last line opposite his side, he is in the so called king's row, and this man is changing into a king and is marked by placing an additional piece underneath. This king now has got additional powers including the ability to move backwards, jump over more

than one empty square and capture forwards and backwards. As with a normal man, a king may make successive jumps in a single turn provided that each jump captures one or more opponent man or king.

Strategy



Players create a strategy when they offer up jumps in exchange for setting up the board, so that they jump over even more pieces on their turn. The player without pieces remaining, or who cannot move due to being blocked, loses the game.

Another strategy is, to get as many kings as possible, because these kings have more power to move and jump.

A special Damath-strategy is to collect as many points as possible. So the arithmetic symbols on the white fields are very important. As well is the gaming time limited.

More information and a game example

Damath instruction:

https://www.youtube.com/watch?v=ljbiJcq17tU

https://youtu.be/oHPvcUaz2z4

What Mathematical Content can be Learned?

Numeracy

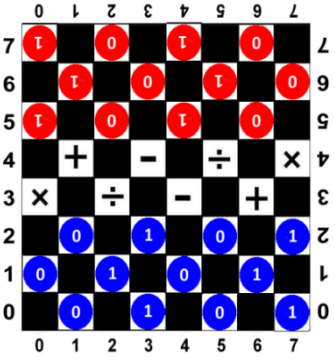
- Read and write numbers from 0 to 11
- Order and compare numbers from 0 to 11
- Do all basic calculations with numbers from 0 to 11, result to 121
- Learn about “0” as a special number in calculations
- Understand the coordinate system in this example

History

The game was invented by a teacher from the Philippines, Jesus L. Huenda, who wanted his pupils to learn Mathematic faster and with more joy like playing games.

Variants

Damath 0; 1

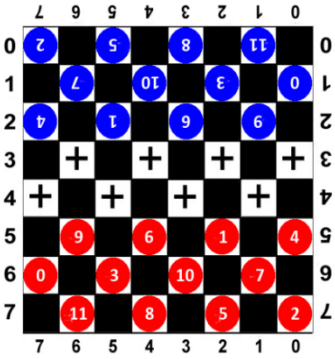


For beginners you can modify the board to work with less value.

Example1: You use just the pieces with the value 0 and 1. The game has just the calculations and moves: 1+1=2; 1+0=1; 0+1=1, 0+0=0; 1-1=0; 1-0=1; 0-0=0; 1×1=1; 1×0=0; 0×1=0; 0×0=0; 1÷1=1; 0÷1=0

0-1 (this move is not allowed, because negative numbers are not known)

1÷0 and 0÷0 (this move is not allowed, because you cannot divide by 0)



Example 2: You can modify the board into a board with just one sort of calculation, for example addition, and all numbers. So you can exercise the addition with numbers from 0 to 9, like 2+5=7 or 9+3=12.

Basic knowledge for all modifications is the ability to play checkers properly.

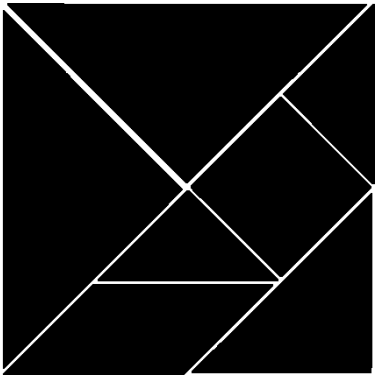
References and Links

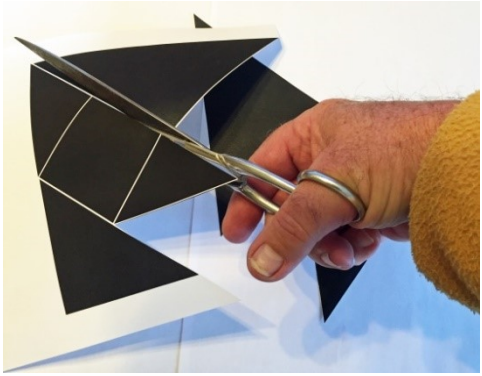
Wikipedia: https://en.wikipedia.org/wiki/Damath

Brettspielnetz: http://www.brettspielnetz.de/spielregeln/dame.php

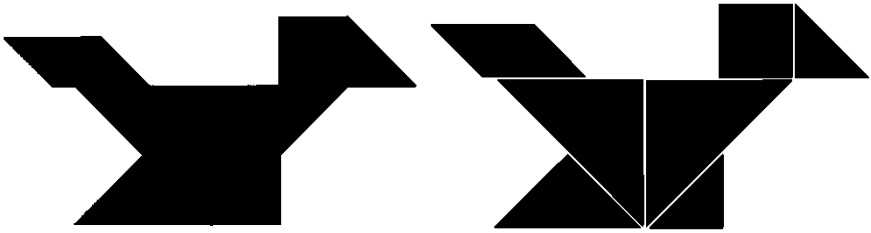
http://download.cnet.com/Damath/3000-18516_4-10911683.html

1.4 TANGRAM





The tangram is a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape, when given only an outline or silhouette (left picture), using all seven pieces, which may not overlap. The solution is shown on the right picture below.



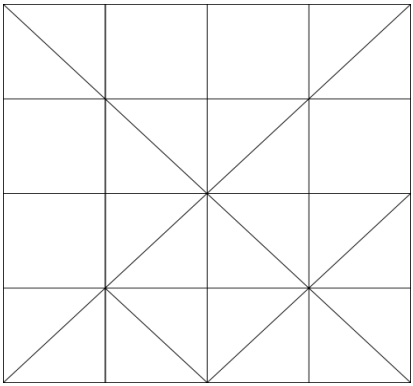
OVERVIEW

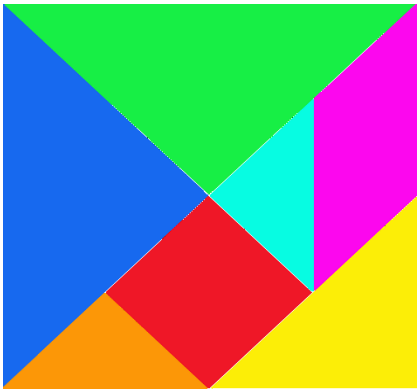
- **Genre:** dissection puzzle
- **Players:** 1
- **Age range:** older than 3
- **Setup Time:** less than 1 minute
- **Playing Time:** as long as people want
- **Random Chance:** none, no dice needed

GAME RULES

GENERAL

The tangram is a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape, when given only an outline or silhouette, by using all seven pieces, which may not overlap. It is reputed to have been invented in China during the Song Dynasty, and then carried over to Europe by trading ships in the early 19th century. It became very popular in Europe for a time and then again during World War I. It is one of the most popular dissection puzzles in the world. A Chinese psychologist has termed the tangram "the earliest psychological test in the world", albeit one made for entertainment rather than analysis. In reality, Tangram is the best and most pleasurable way to learn about geometric shapes like squares, rectangles, triangles and parallelograms.





START

If you don’t have the seven tangram tans, you can make it by your own. The construction is easy and you can construct a coloured tangram as well.

Puzzle

You get an outline or silhouette, like the duck, and you have to form the given shape by using all seven pieces, which may not overlap. The first exercise is always to lay out the stones into a square, which is not easy if you have no pattern.

More information and more shapes

YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Tangram instruction: https://youtu.be/2u_O4fByDgY

What Mathematical Content can be Learned?

Geometry

- Recognize and name two dimensional shapes
- Understand common everyday names of shapes
- Describe length and width of shapes
- Compare different two dimensional shapes
- Understand symmetry in shapes
- Learn about 90 and 45 degree angles
- Paint and construct geometric shapes
- Characteristics of square, parallelogram, right triangle
- Sum of angles in a triangle and rectangle

History

The tangram had already been around in China for a long time when it was first brought to America 1815 by the trader Captain M. Donnaldson. When it docked in Canton, the captain was given a pair of Tangram books from 1815. They were then brought with the ship to Philadelphia, where it docked in February 1816. The first Tangram book to be published in America was based on the pair brought by Donnaldson.

The puzzle was originally popularized in China. A fictitious history of Tangram claimed that the game was invented 4000 years prior by a god named Tan. The

book included 700 shapes, some of which are possible to solve.

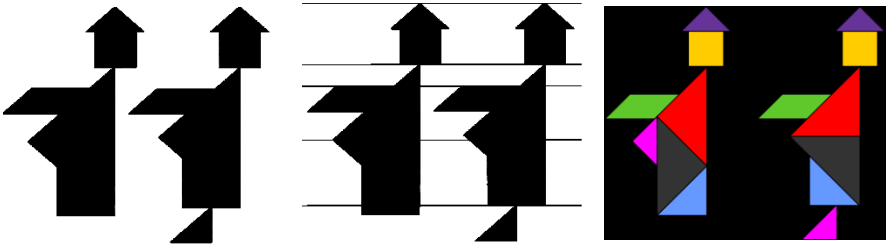
Cover art from The 8th Book of Tan, by Sam Loyd, a spoof of the puzzle's history that began the Tangram Craze in the Western World.

The puzzle eventually reached England, where it became very fashionable indeed. The craze quickly spread to other European countries, especially Germany and Denmark. This was mostly due to a pair of British Tangram books, *The Fashionable Chinese Puzzle*, and the accompanying solution book, *Key.*

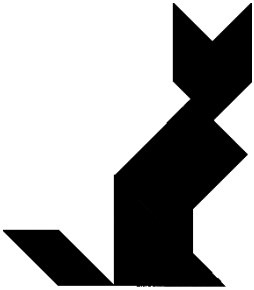
VARIANTS

PARADOXES

You can play tangram in a lot of ways, but most interesting are the “paradoxes”, which means, that two identical shades can be put together in a different way.



Example: The two monks’ paradox – two similar shapes but one missing a foot. The paradox shape (left picture) consists of two similar shapes, one with and the other missing a foot. In reality, the area of the foot is compensated for in the second figure by a subtly larger body.

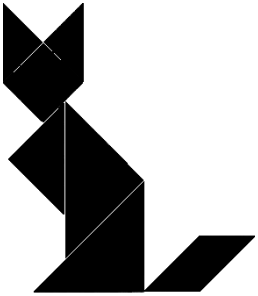


REFERENCES AND LINKS

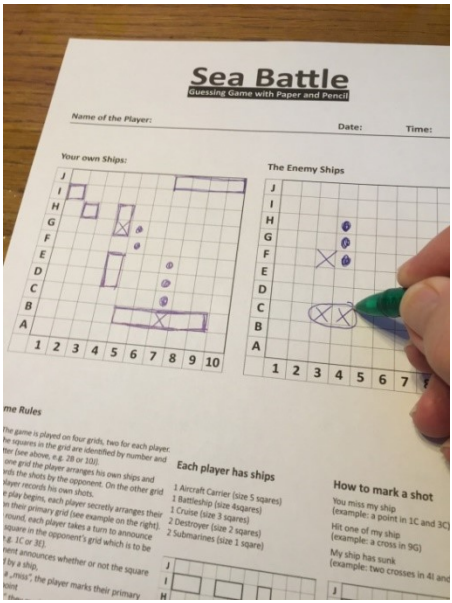
Wikipedia:
https://en.wikipedia.org/wiki/Tangram

Examples:

http://paul-matthies.de/Schule/Tangram



1.5 Sea Battle (Battleship)



The game "Battleship" or "Sea Battle" is a game, which was played by generations of students in boring lessons with their neighbours. One needs only a piece of paper and you're able to play. It sounds like it is a war game, but it is just a guessing game, in which people can learn and use coordinates.

Overview

- **Genre:** Guessing game
- **Players:** 2
- **Age range:** older than 10
- **Setup Time:** less than 1 minute
- **Playing Time:** about 10 Minutes
- **Random Chance:** none, no dice needed

Game rules

Preparation

The game is played on four grids, two for each player. The squares in the grid are identified by number and letter (see above, e.g. 2B or 10J).

On one grid the player arranges his own ships and records the shots by the opponent. On the other grid the player records his own shots. Before play begins, both players secretly arrange their ships on their primary grid:

Each player has 7 ships in his fleet:

1 Aircraft Carrier (size 5 squares)

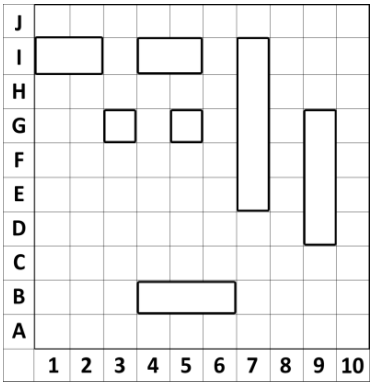
1 Battleship (size 4 squares)

1 Cruisers (size 3 squares)

2 Destroyers (size 2 squares)

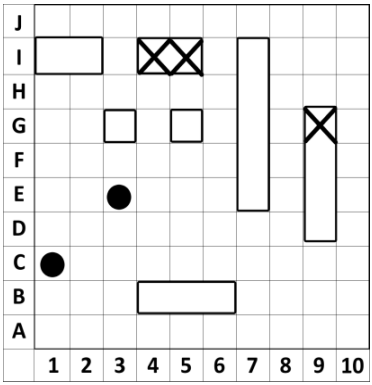
2 Submarines (size 1 square)

(All together 18 squares.)



GAMING

In each round, both players take a turn to announce a target square in the opponent's grid which is to be shot at (e.g. 1C or 3E). The opponent announces whether or not the square is occupied by a ship,



- and if it is a miss, the player marks his primary grid with a point
- if it is a hit the player marks this on his own primary grid with a cross.

The attacking player notes the hit with a cross or a miss with a point on his own tracking grid. When all of the squares of a ship have been hit, the ship is **sunk** (see 4I and 5I), and the ship's owner announces this (e.g. You sank my destroyer!). If all of a player's ships have been sunk, the game is over and their opponent wins.

How to mark

- You miss my ship - Example: a point in 1C and 3C
- You hit one of my ships - Example: a cross in 9G
- My destroyer has sunk - Example: two crosses in 4I and 5I

MORE INFORMATION

YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 18, later to 100

Geometry

- Grid 10 by 10
- Recognize and name two dimensional shapes (square, rectangle)
- Understand a coordinate system and find position up to 10 by 10
- Understand different coordinate designation
- Measure areas by counting squares or using grids

History

Nobody knows exactly, where this game comes from. In a lot of countries the game was played by students since about 1890. You find this game in all countries in different versions, but mainly the name is like “Sea Battle” or “Battleship”.

Variants

There are hundreds of variations possible. Example: If you hit a ship, you are allowed to do a second shot immediately. Find some variations on your own and have more fun with the game.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/Battleship_(game)

Free games to play with the Computer:

http://de.battleship-game.org/

http://www.knowledgeadventure.com/games/battleship/

http://www.mathplayground.com/battleship.html

Instruction and paper to play on (copy the next page)

Sea Battle

GUESSING GAME WITH PAPER AND PENCIL

Task: Copy this page and cut it into 2 pieces. Now you can start playing “Sea Battle”. Please watch the rules!

J										
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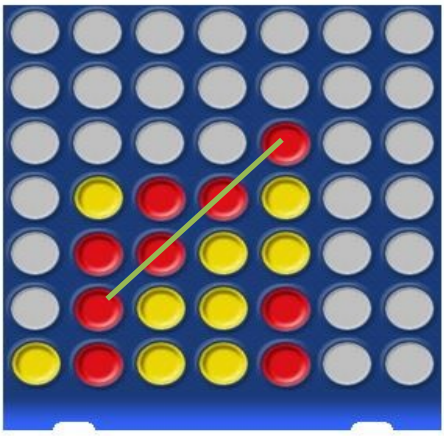
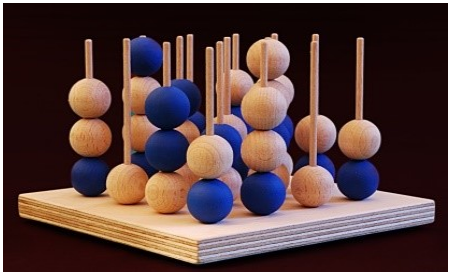
1.6 Connect Four (Captains's mistress)



The game "Connect Four" or "Captain's mistress" is a game, which is a favourite for young and old.

You can play it in two dimensions: as a 2-dimensional-game or more complicated as a 3-dimensional-game.

Overview



- **Genre:** Strategy Game
- **Players:** 2
- **Age range:** older than 4
- **Setup Time:** some seconds
- **Playing Time:** from 1 to 3 minutes
- **Random Chance:** not at all

Game rules

Connect Four (also known as Captain's Mistress, Four Up, Plot Four, Find Four, Fourplay, Four in a Row and Four in a Line) is a two-player connection game in which the players first choose a colour and then take turns dropping coloured discs from the top into a seven-column, six-row vertically suspended grid. The pieces fall straight down, occupying the next available space within the column. The objective of the game is to connect four of one's own discs of the same colour next to each other vertically, horizontally, or diagonally before your opponent.

What Mathematical Content can be Learned?

Numeracy

- Count up to 4

Geometry

- Grid 7 by 6
- 4 in a line (4 in a row)
- Difference between 2-dimension and 3-dimension

Mathematical solution

Connect Four is a two-player game with "perfect information." This term describes games where one player at a time plays, players have all the information about moves that have taken place, and all moves that can take place, for a given game state.

One measure of complexity of the Connect Four game is the number of possible games board positions. For classic Connect Four played on 6 high, 7 wide, grid, there are 4,531,985,219,092 positions for all game boards populated with 0 to 42 pieces. Connect Four is a strongly solved game. The first player can always win by playing the right moves. The game was first mathematically solved by James Dow Allen (October 1, 1988), and independently by Victor Allis (October 16, 1988). Allis describes a knowledge based approach, with nine strategies, as a solution for Connect Four. Allen also describes winning strategies in his analysis of the game.

History

Milton Bradley first sold the 2-D-game under the famous Connect Four trademark in February 1974.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/Connect_Four

Free games to play with the Computer in 2-dimensions:

http://www.gamesbasis.com/vier-gewinnt.html

http://www.lojol.de/html/4gewinnt.html

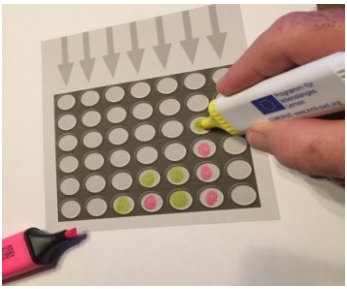
http://www.coolmath-games.com/0-connectfour

http://www.coolmath-games.com/

Free games to play with the Computer in 3-dimensions:

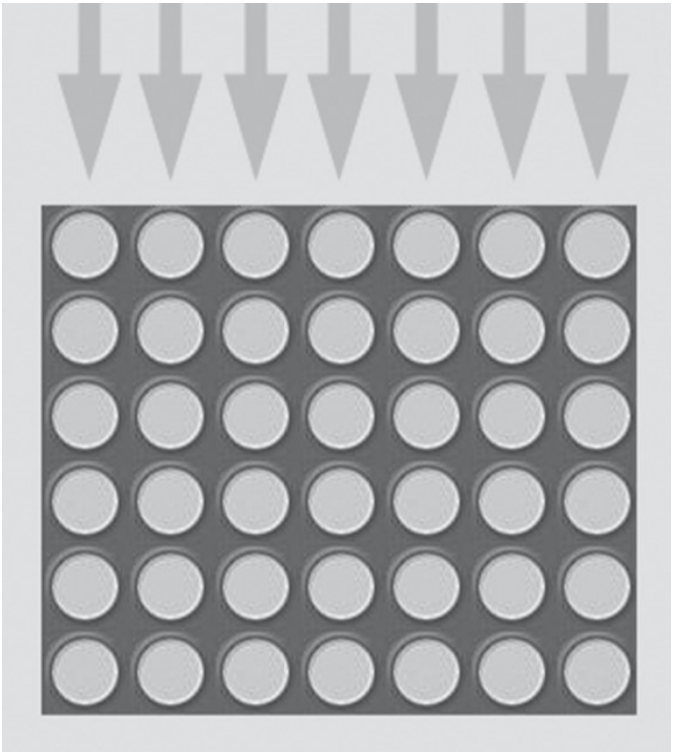
https://cool-web.de/onlinespiele/denkspiele/3d-4gewinnt/3d-4gewinnt.htm

PAPER VERSION



You can play this game with a gridded paper and coloured pencils as well, just watch the rules.

There are some free computer versions (see References and Links) – but it is more fun to play with your friend (copy and play).

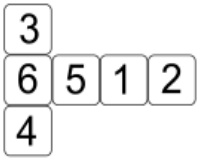


1.7 Ten Dice Games

Dice games are games that use one or more dice as their sole or central component. A dice tray, a tray used to contain dice, is sometimes used for gambling or board games, in particular to allow dice throws which do not interfere with other game pieces.

What Mathematical Content can be Learned?

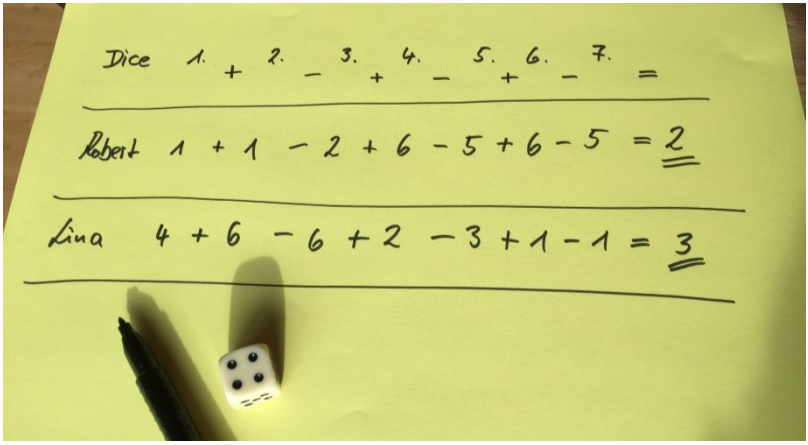
The traditional dice (you can say “one die” or “one dice” or “three dice”) is a cube. Each of its six sides is indicated with a different number of points symbolizing the numbers from 1 to 6. The sum of the numbers of two opposite sides must be 7. The four sections of the cube are regular hexagons, if these sections pass through the centre of the cube perpendicular to the four diagonals. The cube is a special case of the cuboid, prism and the rhomboid. It is perfectly symmetrical and is therefore one of the five Platonic solids in geometry: polyhedrons. The dice-cube is a stereo natural figure with 6 square faces and 12 edges of the same length.



GAME 1: SWING

- You need 1 dice, a list and at least 2 players.
- Each of them rolls 7 times consecutively.
- The following sums and differences are formed: 1st and 2nd throw are added and then alternating each throw is subtracted or added. The highest total wins.
- When subtracting the smallest difference is 0.

Example



GAME 2: NAKED SPARROW

- You need 1 dice, a list and at least 2 players
- You roll the dice and the partner on the left gets the number of the dots as a bonus.
- But if you throw a 1, called “naked sparrow”, then you get a minus point yourself.
- After 4 rounds the winner is the one with the highest sum.

GAME 3: THE EVIL THREE

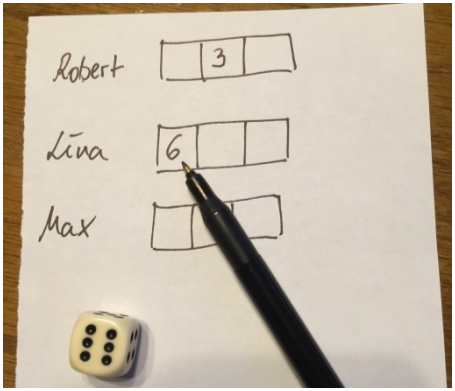
- You need 1 dice, a list and at least 2 players
- Everyone is allowed to roll the dice as many times as he wants and the dots (pips) are summed up and noted.
- However, if you throw a 3, the “evil three”, you are out of the game completely and your sum is 0.
- The highest number of dots is the winner.

GAME 4: SIX MATCH

- You need 1 dice, a list and at least 2 players
- You draw a schedule with six numbered lines.
- The number of columns is equal to the number of players.
- When rolling the dice, the number is multiplied with the line number in which you want to register your throw.
- In every line one may enter only once.
- After throwing 6 times the column is full and the game is over.
- The highest sum of the column wins.

Example: If you throw with your dice a “4” and you want to put it into the 3rd row you have to calculate 4*3=12. You write the 12 in your column/row.

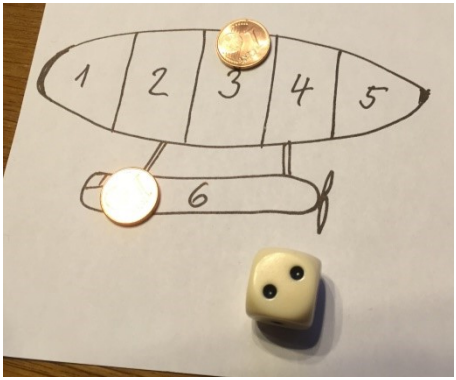
Game 5: House Number



- You need 1 dice, a special list and at least 2 players.
- You draw a list with empty 3-digit house numbers.
- You throw the dice and write the number in a three-digit-list
- You have to do this immediately after you have thrown the dice.
- After 3 throws the highest three-digit-house number is the winner

Example: Robert's first throw was a "3". He put it in the tens digit, Lina throws a "6" and puts it in the hundred digit place.

Game 6: Zeppelin Game



- You need 1 dice, a paper with a painted Zeppelin with 6 pitches, 20 chips or 1-cent-coins and at least 2 players.
- You roll the dice and according to the number you put one chip in the numbered pitch.
- If there is already a chip in the pitch, you can take it.
- Whoever rolls a 6 puts his chip into the gondola. Here you are not allowed to take out a chip.
- Those who have no more chips are out of the game.
- The winner is the remaining player; he owns all the chips from the gondola.

GAME 7: 66 BUT NO 6

- You need 1 dice and at least 2 players.
- Every player rolls the dice as many times as he wants.
- He must sum up the thrown numbers.
- If it is at or above 66, he wins immediately.
- But if he throws a 6, he loses the whole sum and has to start again.
- If he makes a voluntary break, he can go on with the number which he has summed up previously.

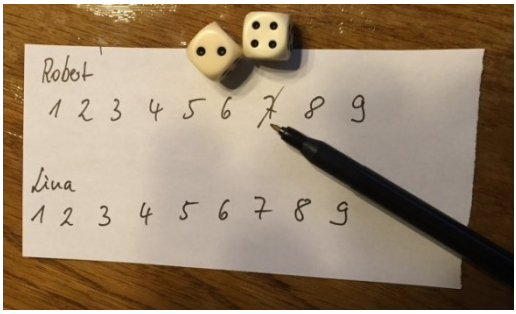
GAME 8: HEAVEN AND HELL

- You need 1 dice and at least 2 players.
- Heaven is the number on top of the dice; it is the tens digit of a two digit number.
- Hell is the number which lies on the table; it is the one digit part of a two digit number.
- The highest number wins.

Examples:

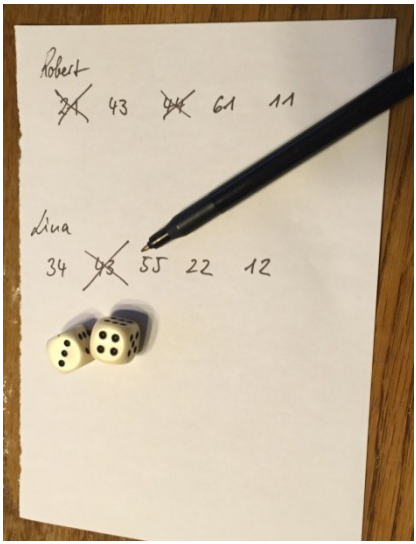
- If you throw a “5” the number on the opposite is “2”, so the final number is “52”
- If you throw a “2” the number on the opposite is “5”, so the final number is “25”
- If you throw a “3” the number on the opposite is “4”, so the final number is “34”. And so on…
- Remark: The sum of the opposite numbers is always 7.

Game 9:
Crossing Out Numbers



- You need 2 dice, a pencil and a paper with a list of numbers from 1 to 9 and at least 2 players.
- You throw two dice at once and cross out the number from the list.
- It is possible to sum up the two dice numbers, to cross out 7, 8, and 9.
- If you have crossed out 7, 8 and 9 you can go on only with 1 dice.
- The one who has crossed out all his numbers from 1 to 9 is the winner.

Game 10: Lucky with Dice



- You need 2 dice, a pencil and a paper and at least 2 players
- You make a list of five 2-digit-numbers from 11 to 66, only with the numbers from 1 to 6.
- You throw two dice at once as long as both dice show one written 2-digit-number. You can cross out this number.
- The winner is the one who has first crossed out all his five 2-digit-numbers.

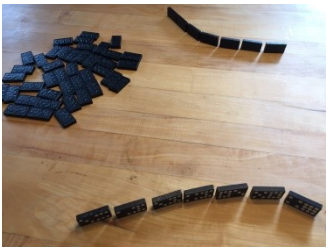
1.8 Domino

Overview

- **Genre:** Tile based game
- **Players:** 2 to 4
- **Age range:** 5 plus
- **Setup Time:** 1 min
- **Playing Time:** 10 to 20 minutes
- **Random Chance:** little, mostly you need tactics or strategy

Domino (Dominoes, Dominos) is a very famous game in a lot of countries. It is played with rectangular "domino" tiles. The domino gaming pieces make up a domino set, sometimes called a deck or pack. The traditional domino set

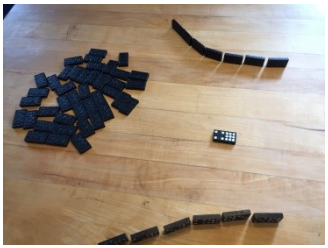
consists of 28 dominoes from 0 to 6 or in some variants of 55 tiles from 0 to 9, colloquially nicknamed bones, cards, tiles, tickets, stones or spinners. Each domino is a rectangular tile with a line dividing its face into two square ends. Each end is marked with a number of spots (also called nips or dobs) or is blank. The backs of the dominoes in a set are indistinguishable, either blank or having some common design. A domino set is a generic gaming device, similar to playing cards or dice, in that a variety of games can be played with a set.



Game Rules

Block Domino

Step 1



The Block Domino game for two players is the simplest basic domino variant. It requires a double-nine set, from which each player gets seven tiles.

The remaining tiles are in the stock, which is only used, if one player cannot put a tile.

Step 2

The first player places a tile on the table which starts the line of play. The second player extends it with one matching tile at one of its two ends.

Step 3

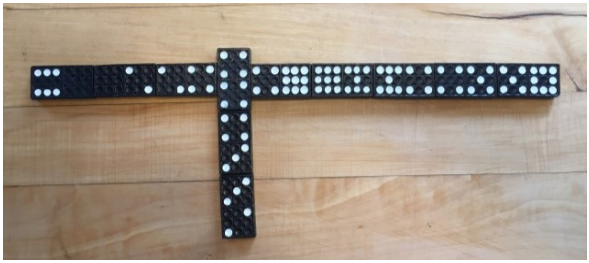
A player who cannot do this passes has to take a new tile from the stock as long as he can put one.

Step 4

If one player puts a double tile (e.g. a double-4 in the picture), he begins a “cross” and put a second tile.

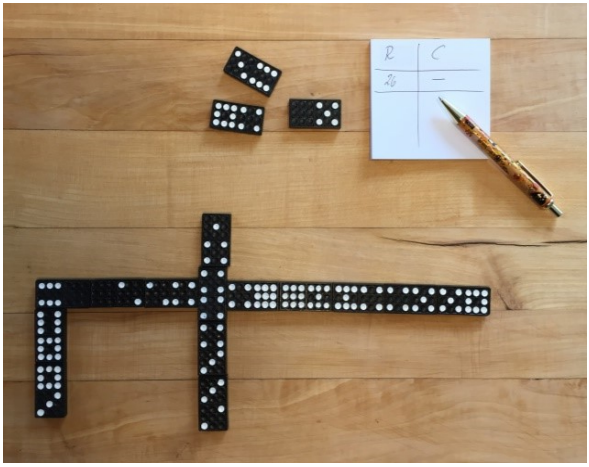
Step 5

The game ends when one player wins by playing his last tile, or when the game is blocked, because neither player can play anymore.



Step 6

The winner's score is the total remaining pip count of the loser's hand. The winner of a blocked game is the player who has a lower pip count, and the score of the game is the difference of the pip counts.



Four Players

There are also variants for four players. The rules are the same, but each player gets in the beginning 5 tiles.

What Mathematical Content can be Learned?

Numeracy

- Count reliably from 0 to 9 items
- Add the points on the tiles
- Recognize points as numbers
- Distinguish different number structures

Variants

For 40 years the game has been played by four people, with the winner being the first player to score 150 points, in multiple of 5, by using 27 bones, using mathematical strategic defences and explosive offense. At times it has been played with pairs of partners. The Double Six Set is the preferred deck with the lowest denomination of game pieces, with 28 Dominoes (tiles or bones).

In many versions of the game, the player with the highest double-leads with that double, for example "double-six". If no one has it, the next-highest double is called: "double-five?", then "double-four?" etc. until the highest double in any of the players' hands is played. If no player has an "opening" double, the next heaviest domino in the highest suit is called - "six-five", "six-four". In some variants, players take turns picking dominoes from the stock until an opening double is picked and played. In other variants, the hand is reshuffled and each player picks seven dominoes. After the first hand, the winner (or winning team) of the previous hand is allowed to pick first and begins by playing any domino in his or her hand.

History

The earliest mention of dominoes is from Song dynasty in China. Dominoes first appeared in Italy during the 18th century, and although it is unknown how Chinese dominoes developed into the modern game, it is speculated that Italian missionaries in China may have brought the game to Europe. The name "domino" is from the resemblance to a kind of hood worn during the Venice carnival.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/Dominoes

YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Instruction for Domino Game (Howcast.com video on youtube):

https://www.youtube.com/watch?v=9MeWPTqJsns

2.1 SKAMBALOVE

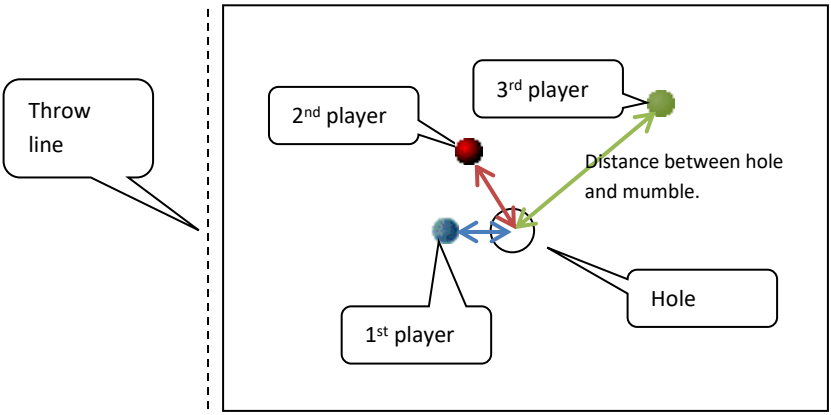
„Skambalove or Skambolove" is a widely spread board race game from the recent past. The rules of the game vary depending on the different regions of the country where it is played. Probably the origin of the game is the medieval game "Marbles". In Bulgaria the rules are transferred by word of mouth without having a clear definition. The version described here used to be played in the city of Kardzhali with glass marbles in the open space. Kids have been playing marbles and marble-like games for thousands of years. It's likely your parents and even your grandparents were playing marbles when they were young.

OVERVIEW

- Genre: marble race game
- Players: two or more.
- Age range: from 7 to 15 years old
- Playing Time: from 10 to 40 minutes (depending on the number of tries and experience of the players)
- Random Chance: none, no dice needed

GAME RULES

The game is played outside in open spaces at a rectangular play area with approximate size 3 x 4 m. A circle hole is dug in the middle of the plain, even surface, without obstacles – with diameter 10-15 cm and depth 5 cm. The hole is usually made by a boot sole. Here you see the play-area:



The marbles (mumbles) used in the game may be made of glass or plastic with diameter around 2, 5 - 3 cm. Each player plays with one marble only, called „Skambalove". The Skambalove is normally a bigger mumble. You need as many "Skambaloves" as there are players here.

Phase 1: Find the playing order

In the beginning of the game the players stand further from the side of the hole in the rectangular. Each of them tosses his marble towards the hole, aiming to move it as close as possible to it or inside of it. The player whose marble is closest to the hole or gets inside of it begins first. Second in turn is the player with the second in distance marble, etc. This rotation order continues until the end of the game.

Phase 2: Play Marbles and Earn Points

After settling the sequence the first task of the players is to place their marbles in the hole. A hit in the hole counts for 3 points. After hitting the hole the player has the right to hit others players' marbles. Each opponent's marble hit counts for 3 points. Once after an opponent's marble is hit, the player has the right to shoot again at opponents' marbles or to place his own into the hole. The winner is the one who first manages to achieve 36 points.

The 33rd and the 36th point cannot be achieved by hitting the hole, but only by two consequent (directly one after the other) hits on opponents' marbles. These consequent hits are named „*Drankel*" and „*Funkel*".

EXAMPLE

https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

https://youtu.be/XgtHVuCDYh8

OTHER RULES

- The player has no right to hit the marble two consequent times in the hole.
- When the player does not hit the hole or does not hit the "*Skambalove*" of another player he loses his turn, so the following player is to play.
- A player, who's marble has left the play area - no matter how - can re-enter the game only after implementing an initial throw followed by a hit in the hole. The points earned until this moment are preserved.
- The implementation of an initial penalty hit is called „*Chorik*".
- The shooting from the hole can be performed from within the hole or from a distance of 4 fingers from the edge of the hole.

GAME VERSIONS

The game has a lot of versions all around the world. One way of playing it is to have a circle drawn in the sand and the players to take turns trying to push the opponents' marbles outside the circle using their own.

Another way is to dig a hole in the sand or in the ground and to try to enter the marble into the hole eliminating the marbles of the other players meanwhile.

In a third version the aim of the game is to hit or capture the opponents' marbles which change their owner by passing from one player to another.

A fourth version is to aim at marbles placed in advance having again the option to aim the opponents' marbles as well.

In order to win the game different tactics are used, such as tossing your marble in a distant and safe zone. As with all children's games the rules vary and adapt to the conditions.

What Mathematical Content can be Learned?

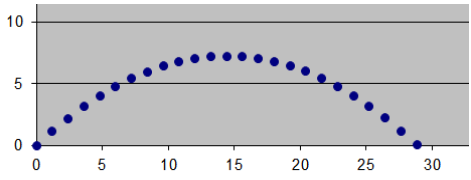
Numeracy

- Counting to 36, by three
- Arrangement and comparison of numbers to 36
- add single-digit and two-digit whole numbers with total to 36

Geometry

- Measure and describe the length between two points (between the mumble and the hole) in centimetres
- Recognize and name two dimensional and three dimensional shapes

Mathematics and Physics



- Coordinate system
- If you throw a marble, the way of the marble is always like a parabola.

Variants

In different parts of Bulgaria the marbles have varying names (in different sources more than 30 variations are mentioned). Many of the names are derivative of the Turkish word *bilyur* (*billûr*), which means „crystal glass“.

The names *Israelis and Syrians* are used for the marbles imported respectively from Israel and Syria. The plastic marbles in Sofia are called *gudi*, respectively the smaller ones – *semigudi* and *minigudi.*

The marbles in Bulgaria are frequently used in math's logical problems. Here is an example:

A math's logical problem for Christmas math's competition in Bulgaria:

There are 5 red, 6 blue and 7 yellow marbles in a bag. How many marbles do you have to subtract at least, with your eyes closed, in order to be sure that you have at least 2 with a different colour?

a) 4 b) 18 c) 8

Solution: Seven marbles of one colour are the most to be subtracted; the eighth will be mandatorily of a different colour.

History

„Skambalove or Skambolove" is a widely spread board race game from the recent past. The rules of the game vary depending on the different regions of the country where it is played. Probably the original of the game is the medieval game "Marbles". We don't know the rules. But probably they played the same basic games of marbles that we know today: one version where you take turns tossing the marbles at a goal (another marble, a hole, or a wall), and another version where you take turns shooting the marbles within a circle drawn in the dirt, trying to get them out of the circle. One version has a series of arches to get the marbles through. The medieval marbles were made of clay, but modern marbles can be substituted by glass or plastic.

In Bulgaria the rules are transferred by word to mouth without having a clear definition. The version described here used to be played in the city of Kardzhali with glass marbles in the open space. Kids have been playing marbles and marble-like games for thousands of years. The game of marbles is played with variations from playground to playground around the world.

References and Links

http://yurukov.net/blog/2007/malkata_radost_dnes/

http://www.peika.bg/statia/Staklenite_topcheta_ot_nasheto_detstvo_kade_kak_se_kazvat_l.a_i.91464.html

https://en.wikipedia.org/wiki/Marble_(toy)

https://de.wikipedia.org/wiki/Murmelspiel

2.2 Не се сърди, човече - Do Not Get Angry, Man (Ludo)

„Не се сърди, човече“.
Ludo is the Bulgarian version of the traditional board game for children and teenagers. This game is often played at home as a family game, but now it has been transformed into outdoors race game versions, as well as into many online game versions. The game board is a closed number of fields in the shape of a cross or a circle. On the line the starting and final flight ("playpen") for the pawns are marked with the corresponding colour.

Overview

- **Genre:** Board game
- **Players:** 2, 3 or 4
- **Age range**: 7+
- **Playing Time:** 30 - 60 min
- **Setup time**: Negligible
- **Random Chance**: Medium (dice rolling)

Game Rules

Two, three, or four players may play. Each player puts four figures in one colour into the small circles of the area (“yards”) in the same colour on the board. Each player rolls the dice; the highest roller begins the game. The next player is in the

clockwise direction. To enter a pawn into play from its staging area to its starting square, a player must roll a 6. If the player has no pawns yet in play and does not roll a 6, the turn passes to the next player. Once a player has one or more figures in play, he selects one of them and moves it forward along the track with the number of squares indicated by the dice roll. The player must always move a figure according to the dice value rolled, and if no move is possible, pass their turn to the next player.

A player may not end his move on a square he already occupies. If the advance of a pawn ends on a square occupied by an opponent's pawn, the opponent's pawn is returned to its owner's yard. The returned pawn may only be re-entered into play when the owner again rolls a 6. A player's home column squares are always safe, however, since no opponent may enter them.

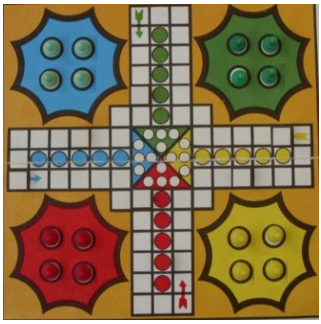
Specific Rules

- If one of the pawns reaches another opponent's pawn, it may depending on the dice roll either skip it or "mount" it.
- If one of the pawns "mounts" an opponent's pawn, the opponent's pawn is returned to its owner's yard.
- When a pawn reaches the centre of the board, it can no longer be "captured".

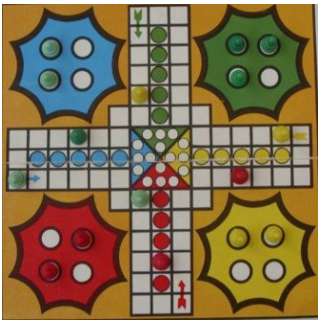
The Game Proceeds in Four Phases

1st Phase: Initial dice roll

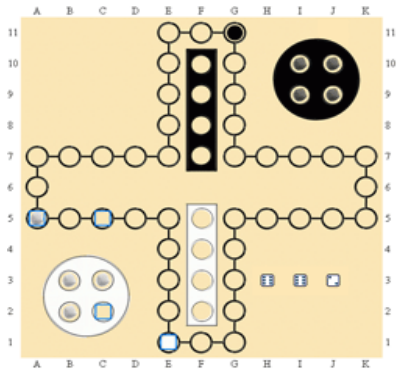
The players begin to roll the dice in alternate turns. When a player rolls a 6 he may choose to advance a pawn already in play, or alternatively, he may enter another staged pawn to its starting square. The rolling of a six the player earns an additional ("bonus") roll in that turn. The rolling of a six earns the player an additional bonus roll again. If the third roll is also a six.

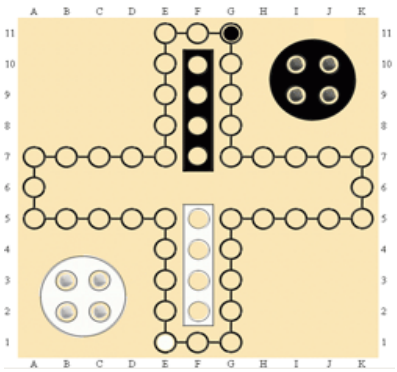


1st Phase: Start



2nd Phase: Moving the pawns



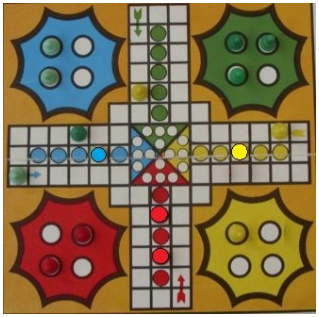


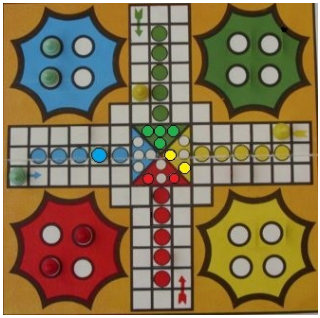
2ND PHASE: MOVING THE PAWNS

Both of the players begin the game with all four pawns in the starting position – the white circle B2, C2, B3, C3 and the black circle I9, J9, I10, J10. Each turn begins with a dice roll followed by movement of the pawns. A roll result of 6 means that the player must roll again and uses two (or more) numbers to move his chosen pawn. The movement on the board is arranged in clockwise direction.

In order to place a pawn in initial position (E1 for the white and G11 for the black), the player must roll 6, and otherwise the pawn remains in the starting point. The picture shows a dice roll 6-6-2, which causes a placement on E1 (first dice roll of 6), followed by movement to C5 (second dice roll of 6) and movement to A5 (dice roll of 2).

During the first turn three rolls are used instead of one in order to quicken the game. In other words the player rolls 3 dices at a turn, until he rolls a 6 and places his first pawn on the board. When this aim is accomplished the following rolls are with one dice only. The pawns are to „pass" up to 56 points on the board until they reach the final corridor.





3rd Phase: Final corridor

4th Phase: Final triangle

3rd Phase: Crossing the final corridor

The final corridor consists of a square with four different coloured triangles. After crossing the final corridor the player begins to arrange his pawns one by one in the final triangle corresponding to the colour of his pawns.

4th Phase: Arranging the pawns in the final triangle

After the player has arranged three of his pawns in the final triangle he must roll the exact dice number to enter his last pawn in the final triangle.

What happens if the players are 4?

When the players are 4 it can be played as a team game, and is only won, when both partners have all eight pieces home. As with all true team games, working together is the key of winning. The complex racing games combine both luck and skills. The players have 4 pawns each and must choose the turns that would be of best benefit for the team.

What Mathematical Content can be Learned?

Numeracy

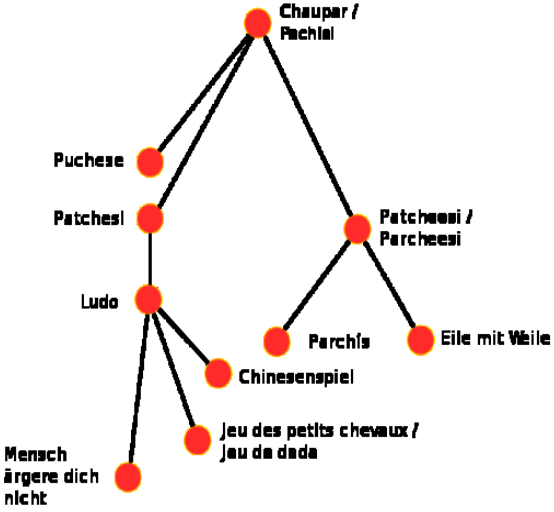
- Count reliably up to 10 items
- Add single-digit numbers with total to 20
- Find halves and quarters of small numbers
- Introduction of two-digit numbers
- Combinational skills
- Coordinate systems
- Counting to 60 – this is the total points' number on the board

Geometry

- Recognize and name two dimensional shapes (circle, square)
- Cube
- Right angle
- Hexagon
- Triangle
- Work out simple volumes

History

The origin of “Ludo” or “Man, don’t get angry!” is Pachisi (also known as Twenty Five). Pachisi is the National Game of India. The name comes from the Indian word "pacis" which means 25, the highest score that could be thrown with the cowry shells. Pachisi is a cross and circle board game. It is played on a board shaped like a symmetrical cross. In 1896, a westernised version of Pachisi was published in England under the name Ludo (Latin for "I play"), a game which has been popular in that country ever since. The game however, is a simplistic version of Pachisi for children. In America, an analogous simplified version appeared called Parcheesi which was for decades, Selchow & Richter's bestselling game. An adapted version was created by Josef Friedrich Schmidt in 1914. In the GDR it was distributed under the name "Raus" ("Get out of here"). In 2014 "Schmidt Spiele" creates a card game version. In Bulgaria “Man, don’t get angry!” has been a popular game during the past century. The rules of the game have not changed until today.



More Information and Game Examples

http://freeigri.com/boardgames/2d-board-games/86-ne-se-sardi-choveche

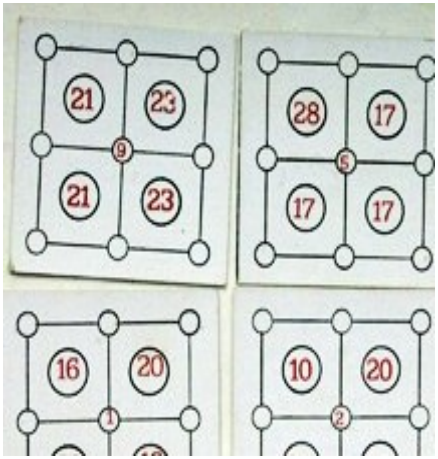
http://us.wow.com/wiki/Ludo_%28board_game%29

2.3 Combination Nine

„Combination 9” is a game that used to be played in the recent past in Bulgaria. It was produced and distributed by CCU-NCC „Happiness”, Sofia

Overview

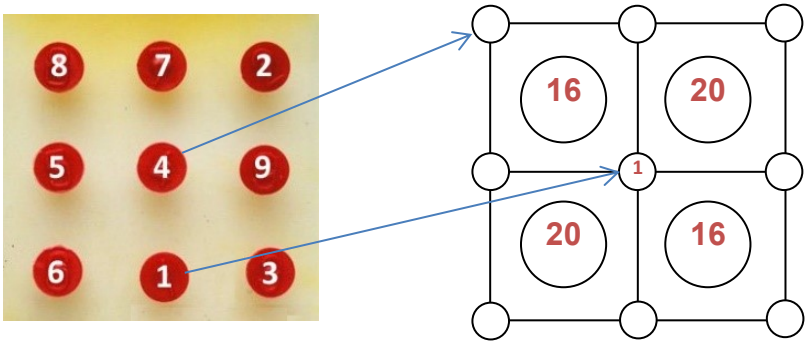
- **Genre:** logical math game, puzzle type
- **Players:** from 1 or 2 players to 2 teams (with random number of players)
- **Age group:** 7 +
- **Duration:** indefinite
- **Choice:** random based only on card shuffler



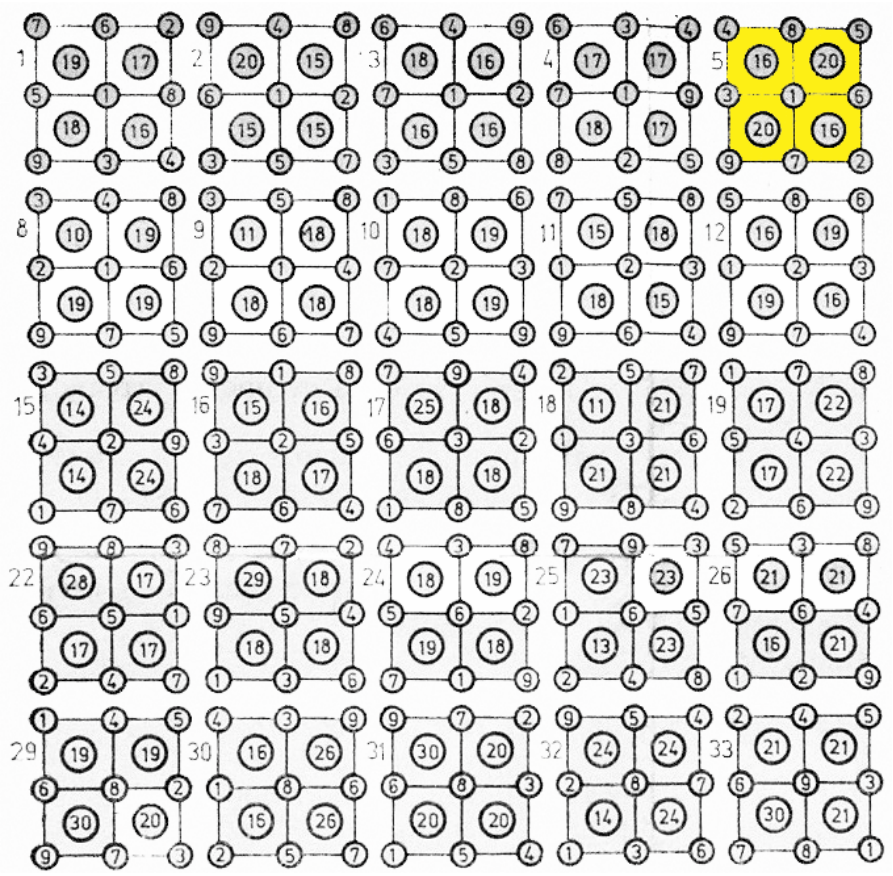
Game Rules

Game set

The game set includes - two square boards with nine nests placed on them, nine green and nine red pawns numbered from 1 to 9 and 35 cardboard squares with height and width 10.4 cm (carnets). Upon them a scheme of 9 circles placed on the edges of 4 similar squares is drawn. Within the circle situated in the centre of the carnet a random digit from 1 to 9 is placed (so-called condition-number), and within the squares a random two-digit number up to 30 is placed (so-called task-number). The circles correspond to the nests on the game boards. After all the carnets are shuffled, the players take a carnet and place it on one of the game boards aiming to arrange the number in the central circle (condition-number), to place the numbers in the rest of the circles on the edges of the squares, but considering the sum of the digits around each square to be equal to the digit inside of it (task number)



Task: Place the numbered pawns to the one selected carnet – the sum of the four numbers must be equal to the number in the big circle! Try as long as all numbers are put to the carnet.



Solution of some examples of carnets

SPECIFICATIONS

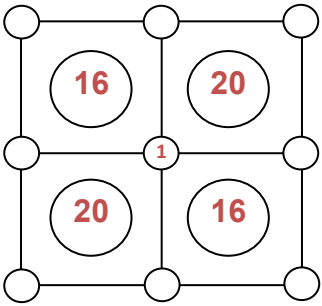
The act of playing may follow different patterns.

- In a series of from 5 to 10 tasks for a previously defined period of time.
- Options for beginners and advanced players with defining of more than one initial numbers.

All together there may be 9! = 362880 carnets. By reducing the carnets which are made by central and axis symmetric, the combinations are 45 360. This makes the game suitable for realization as a computer game, considering that the digital variations may include full combination of the carnets.

THE GAME PROCEEDS IN TWO PHASES:

- Random choice of carnet
- Solution of the carnet

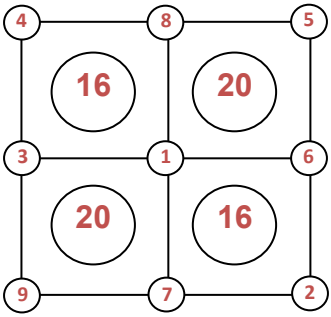


1st Phase: Random choice of carnet

One of the players shuffles the carnets and the participants in the game take them without looking. Each player places the carnet that he took on the game board.

2nd Phase: Solution of the carnet

The game begins when all the players place their numbered pawns in such combination that their sum equals the number in the centre of each of the four squares. The winner is the player who first fills correctly his carnet.



MORE INFORMATION ABOUT THE GAME

http://detstvoto.net/index.php?newsid=2101

What Mathematical Content can be Learned?

Numeracy

- Summation of single-digit numbers and two-digit numbers up to 30 and adding numbers, items or amounts for producing a sum
- read and write numbers up to 30
- Tasks with 4 collectable
- Logical thinking
- Permutation

Geometry

- Right angle
- Recognize and name two dimensional shapes (circle, square)
- Describe length and width of shapes

Statistics

- Applied math – combinatorics

History

There is no information about the history of the game – it was passed from representatives of adult generations and was reproduced as an online game in different versions.

Similar games

Combination 9 is similar to the **Sudoku (see chapter 10.3)** and also to another in Bulgaria and Europe popular game called the **Magic square (see chapter 7.1).**

References and Links

http://sudoku.bg/sudoku-i-teoriqta-na-grafite/

http://mathforum.org/alejandre/magic.square/loshu.html

http://www.taliscope.com/LoShu_en.html

3.1 BLACKJACK

OVERVIEW

- **Genre:** gambling card game
- **Players:** 1 has the bank, 6 more players possible
- **Age range:** older than 18
- **Setup Time:** almost no setup time, you play on special tables
- **Playing Time:** 3 minutes to 1 hour
- **Random Chance:** big gambling chance, because each card is different

GENERAL

Blackjack is a popular American casino game, now found throughout the world. It is a banking game in which the aim of the player is to achieve a hand whose points total closer to 21 than the banker's hand, but without exceeding 21.

The following outline explains the basic rules of standard blackjack (21), along with the house rules most commonly featured in casinos. Players should bear in mind, though, that blackjack rules vary from casino to casino, and check for local variations before playing.

To learn the game you can play free on-line in the following link
http://wizardofodds.com/play/blackjack/

GAME RULES

EQUIPMENT

Blackjack is played with an international 52-card deck without jokers. Casinos normally use several decks mixed together (called a shoe), both in order to speed up the game (with more cards in play you don't have to reshuffle after every

single hand) and to make card counting more difficult (card counting is a technique used to gain an advantage over the casino by keeping track of the proportions of different value cards remaining in play.)

The number of decks differs from casino to casino, but there can be anything from two to eight decks in the shoe. The re-shuffling frequency also differs from casino to casino, and of course depends on the number of decks used. Frequent re-shuffling is another way to diminish the value of card counting.

Casino game rules: The betting limits should be clearly posted on a sign on the blackjack table. Normally, some of the most important rules, such as "Blackjack pays 3 to 2" and "Dealer must draw on 16 and stand on all 17's" are printed on the table.

To play blackjack you need chips to bet with, and you typically buy your chips directly from the dealer. For house games people buy chips and distribute between friends when they play. Having chips to win or lose makes the game more interesting and increases fun.

The standard denominations for chips, also used in casinos are: white chips = €1, red chips = €5, green chips = €25, black chips = €100

Home game blackjack

When you play blackjack at home, you should take turns being the dealer, to ensure fairness in the game (unless the player who is hosting the game specifically has stated that he or she wants to act as the house the entire game and the other players agree to this.) You can switch dealer every hand, every five hands or whatever you decide. If you're playing with a single deck of cards, the best idea is to re-shuffle after every hand. Of course, you don't need a fancy blackjack table to play the game, but you will need at least one pack of cards and something to bet with - cash, chips or maybe matches.

Betting and winning

Each player at the blackjack table has a circle or box to place bets in. There will always be a minimum bet and a maximum bet for the table. The maximum bet is normally ten to twenty times the minimum bet, meaning that a table with a €5 minimum would have a €50 to €100 maximum. Each player decides how much to bet on a hand before the deal.

- Each hand will result in one of the following events for the player:
- Lose - the player's bet is taken by the dealer.
- Win - the player wins as much as he bet. If you bet €10, you win €10 from the dealer (plus you keep your original bet, of course.)
- Blackjack (natural) - the player wins 1.5 times the bet. With a bet of €10, you keep your €10 and win a further €15 from the dealer.
- Push - the hand is a draw. The player keeps his bet, neither winning nor losing money.

Game objective

Although many players may play in a single round of blackjack, it's fundamentally a two-player game. In blackjack, players don't play against each other and they don't co-operate. The only competition is the dealer.

The aim of the game is to accumulate a higher point total than the dealer, but without going over 21. You compute your score by adding the values of your individual cards. The cards 2 through 10 have their face value, J, Q, and K are worth 10 points each, and the Ace is worth either 1 or 11 points (player's choice).

The deal and "blackjack"

At the start of a blackjack game, the players and the dealer receive two cards each. The players' cards are normally dealt face up, while the dealer has one face down (called the hole card) and one face up.

The best possible blackjack hand is an opening deal of an ace with any ten-point card. This is called a "blackjack", or a natural 21, and the player holding this automatically wins unless the dealer also has a blackjack. If a player and the dealer each have a blackjack, the result is a push for that player. If the dealer has a blackjack, all players not holding a blackjack lose.

The players' turns

After the cards have been dealt, the game goes on with each player taking turns - in a clockwise direction starting on the dealer's left.

First, the player must declare if he wants to take advantage of the side rules (explained below). You can only use the side rules once, when it's your turn to act after the deal.

Then the player can keep his hand as it is (stand) or take more cards from the deck (hit), one at a time, until either the player judges that the hand is strong enough to go up against the dealer's hand and stands, or until it goes over 21, in which case the player immediately loses (busts).

Players can take as many cards as they like, as long as they don't bust.

The dealer's turn

When all players have finished their actions and have either decided to stand or busted, the dealer turns over his hidden hole card.

If the dealer has a natural 21 (blackjack) with his two cards, he won't take any more cards. All players lose, except players who also have a blackjack, in which case it is a push - the bet is returned to the player.

If the dealer doesn't have a natural, he hits (takes more cards) or stands depending on the value of the hand. Contrary to the player, though, the dealer's

action is completely dictated by the rules. The dealer must hit if the value of the hand is lower than 17, otherwise the dealer will stand.

Whether or not the dealer must hit on a soft 17 (a hand of 17 containing an ace being counted as 11) differs from casino to casino. There might even be blackjack tables with different rules within the same casino.

Showdown

If the dealer goes bust, all players who are left in the game win. Otherwise players with higher point totals than the dealer win, while players with lower totals than the dealer lose. For those with the same total as the dealer the result is a push: their stake is returned to them and they neither win nor lose.

Players with a blackjack win the bet plus a bonus amount, which is normally equal to half their original wager. A blackjack hand beats any other hand, also those with a total value of 21 but with more cards. As described above, if the dealer has a blackjack, players with blackjack make a push, while all other players lose.

Blackjack Side Rules

Numerous side rules allow for more intricate betting strategies. These side rules can only be used immediately after the deal, before you take any more cards. You cannot, for example, take a third card and then decide to double down.

The most widely practiced options are explained below:

Insurance

When the dealer's face-up card is an ace, each player gets the chance to bet on whether the dealer has a blackjack or not. This is done before any other player actions. The insurance wager equals your original bet and is used to cancel out the likely loss of this bet. A winning insurance bet will be paid at odds of 2:1, and since you lose your original bet, you'll break even on the hand. Strategy guides tend to advice against taking insurance.

Surrender

If you have a bad hand compared to the dealer's hand (judging from what you can see of it) you can give up the hand and reclaim half your bet. The casino keeps the other half uncontested. You need a really bad hand match-up for a surrender to be profitable, such as 16 vs the dealer showing a 10.

Splitting

When you get two starting cards of the same face value, you have the option to split the hand in two. You place another bet of the same size as the original bet and play on with two hands. (Note that it is legal to split 10-point cards even if they do not form a pair - for example you could split a jack and a king.) When you've decided to split a hand, the dealer immediately deals a second card to each hand. Now, if you're dealt yet another pair, some casinos allow you to split the hand again, while others don't. When you're done splitting, each of your hands will be treated separately, meaning that you will take cards to your first hand until you stand or bust, and then carry on with the next hand. If you split aces, you are dealt a second card to each hand as usual, but you are not allowed to take any further cards (unless you are dealt another ace and split again). All hands resulting from splitting aces remain as two-card hands. If the second card dealt to a split ace is a 10-point card you do not receive the blackjack bonus for this hand. It does however win against an ordinary 21 made of more than two cards. If the dealer also has a blackjack the result for this hand is a push as usual.

What Mathematical Content can be Learned?

Numeracy

- Read, count and add numbers 1-11 up to 21+
- Order and compare sum of numbers up to 21
- Improve memory for number cards
- Understand a logical system of rules with numbers
- Understand value and money, recognize and select notes (money)

Geometry

- Recognize and name two dimensional shapes (circle, rectangle)

References and Links

http://www.conjelco.com/faq/bj.html
http://www.luckyblackjack.com/
www.blackjackinfo.com

3.2 MATH-SCRABBLE

34x2=69-1

OVERVIEW

- **Genre:** Mathematical tile game
- **Players:** 2-4
- **Age range:** 5 and older
- **Setup Time:** 1 Minute
- **Playing Time:** 30 Min., sometimes longer
- **Random Chance:** none

GAME RULES

GENERAL

"Math-Scrabble" or "Number-Scrabble" or "Equate" is a game similar to the **Word Game "Scrabble"**. The whole idea is to **construct equalities** instead of words which are the object of the traditional Scrabble.

INTENTION

The game is built around a game board and involves 2-4 players. Also there are a number of tiles with digits or simple mathematical symbols on them (usually the symbols of the 4 operations and the symbol of equality). Each player has to construct valid equalities on a board, vertically or horizontally, using the tiles that he/she has or ones that are already on the board, like a crossword. Each tile carries certain points and the players are trying to use them in order to collect as many points as possible.

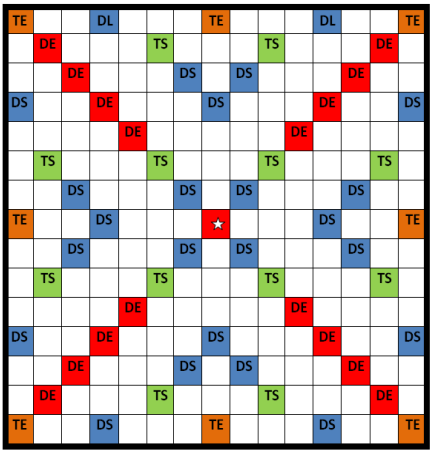
There are various versions of the game but the whole idea is based on the construction of valid equalities vertically and horizontally forcing the other players to minimize theirs and to maximize the points earned at each step.

It is a game that contributes to the realization of the concept of equality and in the comprehension of the basic rules of arithmetic. Furthermore it has the additional value of improving the arithmetical skills of the players through the calculation of the scores according to specified rules.

The game can be extended by adapting the basic equipment or by enriching the rules for the construction of equalities or for scoring.

Objectives of the Game

- To construct equalities in a horizontal or vertical form using the available tiles according to certain rules.
- To count the points corresponding to each equality or equalities formed by a player in his/her last round and add them to form a total score for the player up to this round.
- To continue playing taking turns up to the point when there are no more tiles to be used or the players are not in a position to form any more equalities.
- The player with the highest score wins the game. Thus the Goal of the game is "**to collect the Highest Score**".



Basic Equipment

For the playing of the game the following pieces of equipment are essential:

First a game board consisting of 15X15 squares and having the following form.

Second a number of tiles each representing a mathematical symbol and carrying also a small number at its lower corner corresponding to the value of the points it provides for the score.

Number/ Operator	How many	Score value
1	5	1
2	5	1
3	5	1
4	5	2
5	5	3
6	5	2
7	5	4
8	5	2
9	5	2
0	5	1
+ (add)	7	1
- (subtract)	7	2
× (multiply)	5	2
÷ (divide)	5	3
² (square)	2	3
√(sq. root)	2	3
= (equals)	20	0
blank	4	0
	102	

These tiles may have different distributions in various versions of the game. Also there can be different symbols to be used in according to the version and one can invent even more in order to promote different mathematical skills. As an example, in one of the versions of the game we have the following distribution of tiles (see left).

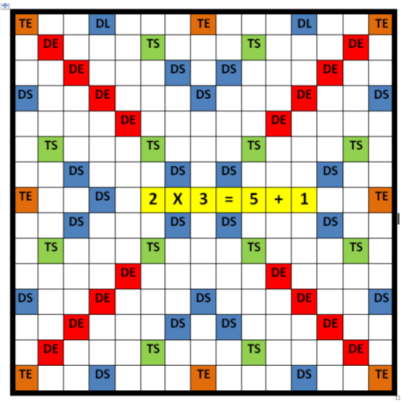
Third you need four racks, one for each player. On these racks each player places his or her tiles so that they cannot be seen by the other players.

Fourth you should have a paper with a table, as can be seen below, to keep records of the score.

Round	Player 1	Player 2	Player 3	Player 4
Round 1				
Round 2				
Round 3				
Round 4				
Round 5				
Round 6				
…..				

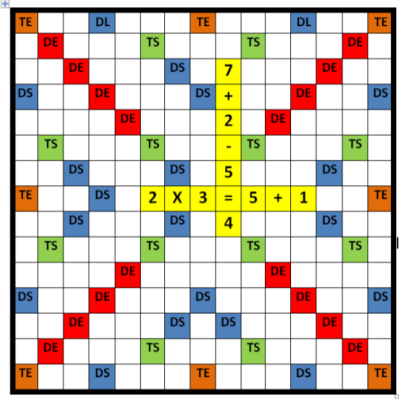
Playing the Game – Processes and Activities in the Game

Preparation



Bring together all the equipment that is needed i. e.

- The board for the game
- The tiles with the mathematical symbols to be used for the construction of equalities
- A cloth bag
- A rack for each player (where he or she lays the tiles that are at his or her disposal) so that they cannot be seen by the others.
- A piece of paper to record the score of each player at each round



How to Begin

- Put the tiles in the bag and shuffle them well. Decide on the priority number (who is the first to play, the second to play and so on) of each player and arrange them around the board clockwise.
- Have each player to pick up randomly from the bag 9 tiles (different to word scrabble), without showing them to the others.
- Each player arranges his or her tiles on the rack in front of him or her so that the others cannot see them.

Playing the Game

- The first player has to lay down on the board equality. The arrangement can be laid using consecutive squares in a row or column (horizontally or vertically) and it should use the central square (labelled with a star).
- Then he/she collects randomly from the bag a number of tiles so that he/she has again 9 tiles.

- If there is no challenge to the validity of the equality the players proceed to the scoring for this round which is credited to the player.
- The game then continues with the next player.

FOR EXAMPLE

The game continues according to following rules:

Players keep 9 tiles in their hand all the times.

The symbol "=" may be used **only once as a new tile** that is placed on the Board.

Equalities may have multiple equal parts e.g.

2x2=5-1=8÷2=4+0

The symbol "-"may be used as an operator for subtraction or a sign for negative numbers.

Optionally the players can decide at the start **to disallow** the validity of some equalities that are considered as "obviously true" e.g. "0+0+0+1=1+0" or "+0=-0"

EXAMPLE

For example at a certain point of the game we might have a table looking like this one (see right).

	9	9	÷	9	9	=	1						
							8	=	7	+	6	-	5
							-						
							1						
					3		0						
				4	×	2	=	5	+	√	9		
2					4		7			4			
6	-	1	=	5	=	3	+	2		=			
-					1		1			8			
6					5					÷			
=					-					4			
2					3								
0													

SCORING

The scoring for each player at each round is kept in a table, usually in the form presented earlier (in the section for the equipment).

Each tile has a certain value of points (which changes with the variation) of the game. Thus adopting the version specified earlier (in the section for the equipment) we have tiles carrying the specified scoring value at the lower right position. Accordingly some of the number-tiles have scoring values ranging from 1 to 4, some of the operator-tiles have scoring values ranging from 1 to 3 and the blank-tile (which can be used as a "joker" in the place of any other tile) has scoring value 0.

For the score of each player calculate the sum of the points corresponding to the tiles used in the equality in the present round, taking into consideration the advantages (extra points) specified at the places on the board. Some of these places specify the calculation of extra points for the player, thus using squares marked as following the score changes accordingly:

Squares marked by	**Benefit in points for the player**
TE	Triple Equality's score
DE	Double Equality's score
TS	Triple Symbol's score
DS	Double Symbol's score

Furthermore if a player uses all his 9 tiles in a single round he is given a bonus of 40 extra points for this round. The scoring continues up to the point when the game ends (as specified below).

ROUND 1:

Equality	**2 x 3 = 5 + 1**	**Comments**
Score due to tiles	1+1+2+1+3+1+1=10	
Score adjusted due to benefits of symbols	1+1+2+1+3+1+2 = 11	There is one symbol on a DS square
Score adjusted due to benefits of equalities	2x11=22	The equality passes through a DE square
Score adjusted due to bonuses	none	
Total score for the round	**22**	

ROUND 2:

Equality	**7 + 2 – 5 = 4**	**Comments**
Score due to tiles	4+1+1+1+2+1+2=12	
Score adjusted due to benefits of symbols	8+1+2+1+2+2+2 =18	There are three symbols on DS squares
Score adjusted due to benefits of equalities	none	
Score adjusted due to bonuses or penalties	none	
Total score for this round	**18**	

In some variations of the game there are also special rules for extra bonus points or penalty points. In the present game we do not consider these.

The game ends in the following cases

- When there are no more tiles in the bag and one player uses the last of his/her tiles. The player that goes out adds to his/her score the total of all the individual scores that the other players are left holding. Also, each player left with tiles must subtract from his/her score the total of the tile scores he/she is left holding.
- When there are no more tiles in the bag and no player has used all his tiles in the last round (i.e. it is impossible for any player to go out). In this case the game ends when each player has one more turn, successively. Each player subtracts from his/her score the total of the individual scores he/she is left holding.

Strategies

Obviously one should adjust his/her game to take advantages of some approaches that will provide the momentum for obtaining the goals of the game. In this sense it is useful to consider some strategies. A player should strive to place his/her tiles on positions providing benefits. These are the positions labelled as TE, DE, TS, DS.

A player should consider of constructing equalities with the maximum possible score, particularly by using all the 9 tiles (which means 40 points bonus). This is not always true, since sometimes the laying of his/her equality might give advantage and solutions to the opponents or even help them to end the game.

Single digits placed adjacent to one another form large numbers

Considering alternatives of constructing equalities usually leads to optimum solutions. For this to have a mathematical background is advantageous.

What Mathematical Content can be Learned?

Numeracy

- Order and compare numbers up to 100, including zero
- Use and interpret +, -, x, / and = in practical situations for solving problems
- Do all basic calculations with single-digit and two-digit numbers from 0 to 100

3.3 MONOPOLY

OVERVIEW

- **Genre:** Mathematical tile game
- **Players:** 2-4
- **Age range:** 5 and older
- **Setup Time:** 1 Minute
- **Playing Time:** 30 Minutes, sometimes longer
- **Random Chance:** yes, you play with a dice

GAME RULES

GENERAL

Monopoly is a game that was developed as an educational tool in order to face the repercussions of monopolies and help the people understand how they have to deal in a competitive market. Actually the word monopoly has its origin in the Greek words ΜΟΝΟΣ (single) and ΠΩΛΩ (to sell) and in this sense reflects the whole idea of a market where the people are competing in order to have the sole share of the market and thus have the possibility of becoming richer and richer. This competitive approach reflects the whole philosophy of capitalism and the game is a miniature of what we actually face in real life.

According to the definition of monopoly in the economic theory it is a situation in which a single company or group owns all or nearly all of the market for a given type of product or service. By definition, monopoly is characterized by an absence of competition, which often results in high prices and inferior products.

The game is built around a game-board and involves 2-8 players that are moving around the board according to specified rules, trading, developing and managing properties. In the context of this process the players are trying to drive the others into bankruptcy and thus win the game.

The game has its origins at the beginning of the 20th century and it has been developed in a number of versions. Further to the reflection of the realities of the market the game has educational value both for the social sciences as well as for the learning of mathematics, In the latter case it can be used to learn basic consumers mathematics (for all ages over 8 years old) as well as more advanced ideas, including mathematical modelling.

In this presentation we describe only the basic ideas of the game, as it is played in a very large number of countries, without expanding to the various versions and variations.

Goal of the game

The game is a competitive effort of the players involved to become as rich as possible and aims at driving all the other players into bankruptcy, leaving one, the monopolist, as the winner thus the game reflects some ideas of economic theories and the effort to control the market by a single person or business.

Basic Equipment

To play the game the following pieces of equipment are essential

The game-board which is a square with 40 rectangles as can be seen in the diagram on the right.

Each of these squares determines a certain role specified by a clear written insertion on them. There are variations on these insertions depending on the version but in the standard form each of these rectangles correspond to

- One of 28 properties (corresponding to 22 streets grouped into 8 colour sets), 4 railway stations and 2 utilities
- 3 chance spaces,
- 3 community chest spaces,
- 1 luxury tax space and 1 income tax space and

- 4 corner squares corresponding to **GO**, **In jail/just visiting**, **Free Parking** and **GO to Jail**.
- 16 **Chance Cards** and 16 **Community Chest Cards**. These are cards that the players draw when they arrive at the corresponding rectangle of the track in accordance to the instructions of the written insertion on it.
- 28 **title deeds** each of them corresponding to each property. These are given to the players as a sign of ownership (when they buy the property). On each of them it is specified the purchase price, the mortgage value, the cost of building houses and hotels on that property and the various rent prices depending on the extent of development of the property is specified.
- A pair of **six sided dice** (in other variations a speed die is added)
- 32 images of **houses** and 12 images of **hotels**
- An amount of **money** (varying with the version) in denominations of ~~M~~ 500, ~~M~~ 100, ~~M~~ 50, ~~M~~ 20, ~~M~~ 10, ~~M~~ 5 and ~~M~~ 1. The total amount in older versions was ~~M~~ 15140, but as the banker in the game is entitled to print more money, it is not of significance.
- 8 tokens (of different colours or shapes) that represent each player in moving around the board and specify his/her position at any moment.

Basic Ideas of the Game

The game can accommodate 2 to 8 players and one person who is called the banker.

The banker is the one that has the responsibility of over viewing and coordinating the management of the money and the properties but he/she can be a player as well.

The players start with an amount of money and by rolling the dice they move around the board. When they arrive at a certain rectangle they are subject to the specified rules.

Accordingly they buy and sell properties, they pay rent and taxes and they get loans on mortgages. In this process they get wealthier or they lose money and property. Some of them become bankrupt and have to leave the game. The one that manages to be the wealthiest wins the game.

The three pictures show an example of a chance card, an example of a community chest card and an example of a title deed.

Playing the Game – Processes and Activities in the Game

Before starting the game the players determine the **banker.** This person can also be a player of the game but in addition he/she has the responsibility to manage the money and the properties. So, initially all the money and properties belong to the bank. If he/she is also a player he/she has to keep what he/she acquires/ owes in the latter capacity separately from what belongs to the bank.

Furthermore, before the game starts, the following preparatory activities take place:

- The board is arranged and the money, the title deeds are set in front of the banker (separately from any material that belongs to this person if He/she is to be a player as well)
- The chance cards and the Community Chest Cards, well shuffled, are put on their reverse sides in the form of decks at the corresponding rectangles on the board

Then the players have to choose **a token** which is the unit representing them when they move around the board.

Each of the players is given a fixed amount of money by the bank (usually ~~M~~1500 in various denominations that have been decided) to be his/her initial assets and which he/she keeps in front of him/her.

In order **to start the game** and decide on who is the first, second and so on they roll the two dice. Whoever rolls the highest sum is the first one and so on. Then they arrange themselves clockwise around the board and they start the game.

According to the outcome of the dice they move their token around the board starting from the rectangle denoted by GO. In case that a player rolls a double he/she is entitled to another turn after he/she performed what follows from the first throw.

In the case of landing on a rectangle with a coloured strip across the top or corresponding to a railway or a utility, the player may **BUY THE PROPERTY** for the amount printed on the board.

As a result of buying the property the banker gives him/her the deed of the property. If the player does not want to buy the property it goes to **auction**. The bidding starts at any amount and the process continue for as long as they can pay. The bidder with the highest bid pays the money to the banker and he/she gets the respective deed.

In the case that someone lands on a property belonging to someone else the owner is entitled to **COLLECT RENT**. The amount for rent is determined by the insertions on the title deed card of that property.

WHAT IS A MONOPOLY

If someone owns all the properties in a coloured group he/she has a MONOPOLY. In the latter case the owner of the monopoly has the following rights:

- He/she may charge double the rent for his/her properties
- He/she can start building houses on these properties he/she charges more rent. The prices for building and extra rent are determined on the title deed of that property. Once you have built **four houses** on a property the owner of it may replace them with a **hotel,** having thus the right to charge more money when someone lands on that property.
- The building of houses on the properties of the same colour should be gradual. Thus the owner of the monopoly for that colour should build one house on each of the properties of the same colour, then a second house on each of them, followed by a third and a fourth on each of them.
- The players are allowed a **SALARY** of ~~M~~ 200 each time they land or pass the rectangle marked by GO.

SPECIAL RECTANGLES

In the case that someone lands on a rectangle marked **Chance** or **Community Chest** he/she is entitled to take a card from the corresponding deck of **Chance or**

Community Chest cards. These cards specify an amount that has to be paid or to be collected by the player. After reading and processing the action specified on such a card it has to be returned to the bottom of the corresponding deck.

Interesting Features

An interesting feature of the game is to be sentenced to land on the rectangle **IN JAIL.** This can take place in the following cases and in any of them he/she is not entitled to collect ~~M~~ 200 when passing the GO rectangle:

- Roll three doubles in a row. In this case the player after the third throw has to move this rectangle without any other action.
- Picking a Chance or Community Chest card with the instruction "Go to Jail"
- Land on the rectangle with the instruction "Go to Jail"
- When someone is **IN JAIL** his/her turn has ended and they have to wait for the next turn to roll dice but meanwhile he/she can continue getting rent, participating in auctions, buying houses or hotels or mortgaging or doing other exchanges.

In order to **GET OUT OF JAIL** i.e. to move out of this rectangle one of the following has to take place on the next turn:

- Pay ~~M~~ 50. When you are out of jail you proceed with the game after this turn by rolling dice. Roll doubles instead of paying ~~M~~ 50. If you fail to roll doubles you have to stay in jail. If this process happens three times you have to pay ~~M~~ 50 and then proceed with the outcome of the dice. Use a card marked "Get out of jail". This might be a card that you have picked up previously from the chance or Community chest cards or a card that you can buy from other players.
- At any time in the game players are allowed to **TRADE** money or property as long as they both agree.
- In case that a person has not enough money to pay rent or other exchange he/she is entitled to **MORTGAGE** his/her houses and get money from the bank. When a house is mortgaged, no rent can be collected. To lift the mortgage for the house the player has to return to the bank the loan plus a 10% interest.
- In case someone has to pay more money that his/her assets allow, he/she is declared **BANKRUPT** and goes out of the game. The last player who remains in the game wins.

Useful Strategies

The players are advised to organize themselves so that they keep records of what they have at any stage in the game. Written records are helpful covering their

- Money in cash
- Income they get from renting or selling properties
- Any other income (e.g. salaries)
- Properties and their value
- Mortgages that they get from the bank
- Payments they spend for buying, renting
- Payments for fines, taxes, interest or any other expenses
- The total value of their assets at any moment has to be at their disposal so that they can decide on their next actions.

What Mathematical Content can be Learned?

The game provides ample opportunities for consideration of the most basic Consumer Mathematics up to more advanced topics that relate to the managing of mathematical models for financial issues. It can be used for simple arithmetical operations, extent to everyday concepts like percentages and provide the forum for more advanced models. It can be used with adults with very limited background in mathematics up to adults with more advanced skills. It can also be adapted for presenting and explaining various ideas of mathematics and its applications starting from Consumer Mathematics and basic ideas about functions and graphs to ideas of Probability and Mathematics of Finance.

Numeracy

- Recognize and select coins and notes
- Make amounts of money
- Calculate with money

References and Links

Philip E Orbanes: Monopoly: The World's Most Famous Game

From Wikipedia: https://en.wikipedia.org/wiki/Monopoly_(game)

How to Play Monopoly: http://www.wikihow.com/Play-Monopoly

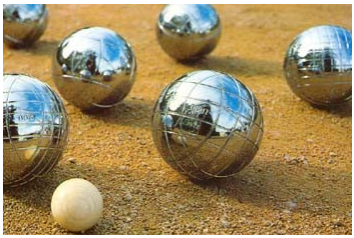
4.1 Petanque

Overview

- **Genre:** boules game
- **Players:** max. 6 and min. 2
- **Age range:** older than 5
- **Setup Time:** 5 minutes
- **Playing Time:** undetermined
- **Random Chance:** none, no dice needed
- **Remark:** Pétanque is a physical game. In the official competitions the players do not sit down and they move a lot in order to prepare each move.

Game rules

General



Pétanque is the most popular of the family of boules games and perhaps the widest spread. In 2007 there were 558 898 licensed players around the world - in more than 78 countries. In France pétanque is the tenth sport by number of licensed players: 311 971 in 2010. We also have to mention that, if pétanque is mostly a masculine game, in France 14% of licensed players are women. Also, it is one of the few mixed games. There is no special women competition, the teams can be mixed.

The game is played by teams of 2 or 3 players. If there are two players, each throws three boules. If there are three of them, each player throws two boules. The game can be played on any type of field. In the beginnings and nowadays pétanque is played outside in open areas, but usually the official competitions are

played indoors. The area used for playing is called “terrain”. For tournament play, a marked terrain is a rectangle at least 4 meters wide and 15 meters long.

The equipment is composed of the boules, the jack and sometimes a plastic circle of 50 cm diameter. The boules are made of metal (usually steel) and have a diameter between 70.5 mm and 80 mm and a weight between 650 g and 800 g. When played by children the boules are made of plastic.

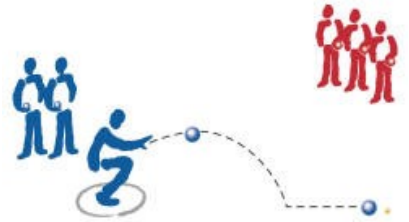
The jack or the “*but”*, the “*bouchon”* – from the occitan *bochon, the “petite boule”,* the “cochonnet”*, “*le petit*”* as it is called in French is a small ball of 30 mm diameter made of wood, usually boxwood or beech wood.

The plastic circle is used to mark the place from where the players have to throw the boules. We have to mention that when throwing the player has to keep his feet stuck to the ground.

The aim of the game is to throw the boules as closely as possible to the jack.

PÉTANQUE STEP BY STEP

Pétanque is not a very complicated game. Once you have caught the overall sense, you start to enjoy watching a game and discovering the strategies. You will also come to understand a little bit of its spirit, of the closeness it brings. Or why French people love to play pétanque in the warm summer days while having a glass of wine. This is the sense of sharing: *la convivialité*.



Learning the rules is easier if you bear in mind that this game is close to basketball or curling.

The steps in playing pétanque:

1. Decide who goes first. Once you have the teams and all the equipment in place, toss a coin to decide which team goes first.

2. Toss the jack. The first team will chose the location of the tossing circle. A player from the first team will step into the circle and will toss the jack. The jack than be thrown in any direction but must land within 6 m to 10 m of the starting circle and three feet from any obstacle.

Boules jack and circle

3. Throw the boules. The first team that tosses the jack has to choose a player who will throw the first boule. It can be the same one who tossed the jack or another one. One has to throw the boule the closest possible to the jack. One can hit the jack but one has to be careful not to take the jack out of the playing space = dead jack. When throwing one has to keep the two feet on the ground and within the circle until the boule touches the ground. Afterwards, one step out of the circle and a player from the second team will take his place. The objective of the second team player is to throw the boule even closer to the jack then the first team player. One can do so also by throwing the opponent out of the way = knocking the first player's boule out of the way. The boule closest to the jack leads or is said to be "holding the point". The other team must continue throwing boules until they take the lead or run out of boules. There is no precise order in which the team members throw the boule. They just have to throw their own boules (not allowed to throw in replacement of a team mate). If the second team manages to take the lead, the first team then tries to recover by landing a leading boule.
4. Winning the Round. Once a team throws all its boules, the other will do the same until there are no more boules to play. The second team to have finished the boules is the winner of the round and in the same time it will gain a number of points according to the number of boules closer to the jack than their opponents' boules. Only one team scores points during a round.
5. Beginning a New Round. Once the points are counted, the next round begins with the previous round's winners drawing a new starting circle. This starting circle is drawn around the final position of the jack in the previous round.
6. Winning the game. The first team to score 13 points wins the game. In doing so there is no maximum number of rounds to be played.

Strategy

As one can notice by now, the game in itself is not very difficult, but even so it is widely popular. Its popularity is due to the quantity of strategy and imagination that lies behind every move and to the spirit of the game that we have already described. This is the reason why no round will resemble another.

With every boule to be played, there are so many different ways of playing the game:

- shoot away your opponent's boule

- obstruct the path to the jack

- change the position of the jack with the boule etc.

The strategy is drawn according to the situation.

More information and a game example

Pétanque instruction (videos on youtube):

https://www.youtube.com/watch?v=YK9EKVuaRU0

https://www.youtube.com/watch?v=5s9BpxOsOY4

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 9
- Read and write numbers up to 9
- Basic calculation
- Calculating probabilities

Geometry

- Line
- Distance measuring
- Right angle
- Recognize and name two dimensional shapes (square, rectangle, circle)

HISTORY

The beginnings of the game can be traced in the Ancient times. It is said that the Greeks in the 6th century BC were playing a game of throwing coins, later stones and round stones called Spheristics. The player had to throw these objects as far as possible. Later on the Romans modified the game in throwing the stones as closely as possible to another object. The Roman variant of the game came to Provence with the Roman soldiers. Later on it was also exported to England were it became really popular and continued to be developed and played until today under different names. It is said the game was so popular that Henry III had to ban it because his archers were playing boules all day long and not practising archery anymore. In Southern France the game evolved into jeu provençal (or boule lyonnaise), similar to today's pétanque, except that the playing area was longer and the players ran three steps before throwing the ball. The game was played in villages all over Provence, usually on squares of land in the shade of plane trees. Matches of jeu provençal around the turn of the 20th century are memorably described in the works of novelist Marcel Pagnol.

According to some documents the game in its current form was created in 1910 by a café owner named Ernest Pitiot in the city of La Ciotat near Marseille. Pitiot wanted to accommodate the old jeu provençal player named Jules Lenoir, whose rheumatism prevented him from running before he threw the ball. For this the length of the field was reduced roughly by half, and a player no longer engaged in a run-up while throwing a ball - he stood, stationary, in a circle. In the same year, 1910, the first official competition was held at Pitiot's café. Petanque spread rapidly from Provence to the rest of France, then to the rest of Europe, and then to French speaking colonies and countries around the globe.

Synonyms

The game is known as pétanque after the French name. There are other games that can be included in the same category of games: boules games. These include bocce in Italy, lawn bowling in England, and bowling in America. The boules games share the same very long history.

Variants

Pétanque is one game in the large family of boules games. We cannot say the following games are variants of pétanque but rather that they are variants of boules games:

- Bocce, the grand-father of boules games. In this game the boules are in wood and it uses a run-up throwing technique.
- Boccia is the bocce game adapted for players who are confined to wheel chairs
- Bocce-volo is like the bocce but it uses metal boules and the run-up is really complicated
- Jeu provençal or boule lyonnaise is like the pétanque but with a larger field and no run-up
- Bowls or lawn bowl. It is known also as the green bowling. It is like the bocce, or the bowling save that it is played outdoors on grass.
- Curling. It is similar to pétanque but on ice. The boules are in granite, there is no jack, there are ten rounds and the players slither when throwing the boule.

References and Links

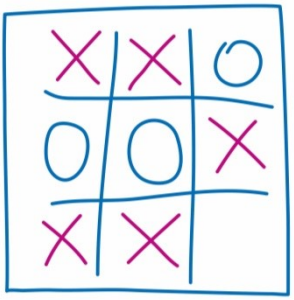
Wikipedia: https://en.wikipedia.org/wiki/P%C3%A9tanque

Play a boule: http://www.playaboule.com/Simple_petanque_rules.aspx

British pétanque: bhpetanque.org

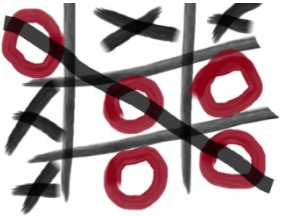
http://www.britishpetanque.org/how%20to%20play.htm

4.2 Tic Tac Toe



Overview

- **Genre:** strategy paper and pencil game
- **Players:** 2
- **Age range:** older than 5
- **Setup Time:** less than 1 minute
- **Playing Time:** from 3 min to 1 hour
- **Random Chance:** none, no dice needed



Game rules

General

TIC-TAC-TOE is a paper and pencil game for two players. One player plays with **X** and the other one with **O**. The players will draw a 3x3 grid. They will take turns in marking the space and the first player that succeeds in placing three of his marks in a horizontal, vertical, or diagonal row wins the game.

If the two play correctly the result will be even. The only way to win the game is to catch your opponent unprepared and attract him/her into a trap. One of the few rules is that the moves can be made in the free square on the right, on the left, up or down. The diagonal move is forbidden. It is also forbidden to jump over a square.

It is also interesting to note that the first player is certain to win if his first move is in the centre of the grid. This is the reason why this first move is forbidden. In consequence, if the players make the rational choice move, the result is certainly a “deuce”.

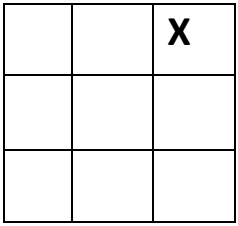
O		
X	O	
X	O	**X**

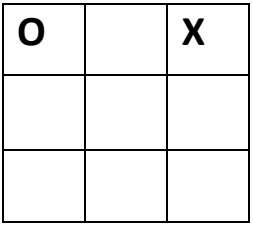
One can notice that no random chance is involved and that the complete information is known by the 2 players - all the moves are known by both players. Do not be fooled! This game seems very simple but it has a lot of traps.

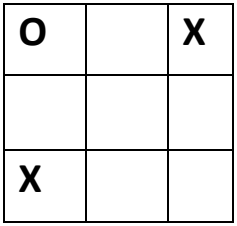
THE GAME PROCEEDS IN TWO PHASES:

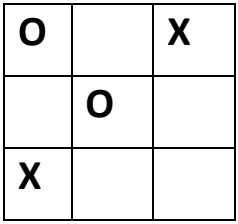
1st Phase: first moves

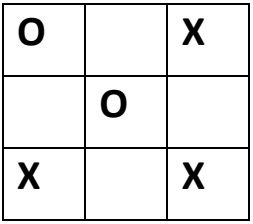
The players will draw a 9 squares grid. They will also determine who plays first and which one of them will play with the X and which one will play with the O. The first player can place **X** or **O** anywhere on the board except in the middle. The second player makes his move in order to block the first or put into motion his one strategy.

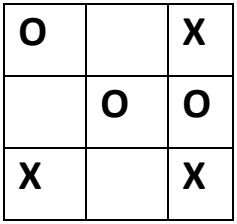


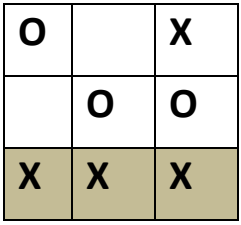












2nd Phase: Moving X and O

Players continue to alternate moves. Each player tries by his moves to anticipate and block the other one's strategy while attempting to achieve a diagonal. They cannot skip squares or move on the diagonal. There are two possible conclusions to the game:

- One of the players makes a diagonal and wins. If the two players have some experience this conclusion is hardly possible because anticipating the other player's moves becomes easier.
- The players run down all the possible moves and no diagonal is made or there are still some possible moves but it is clear that it is no longer possible to make a diagonal.

Strategy

Some people would say this game doesn't seem very difficult because in general it is played by young children. This affirmation is not wrong but that doesn't mean that there is no strategy. On the other hand, one could nonetheless argue that actually it is all about strategy and anticipation. A simple count shows that there are up to 19,683 possible board layouts - 3^9. If we take into account the rules of the game and we assume that X starts each time, it is possible to identify 91 distinct positions that are won by X, 44 that are won by O and 3 that are drawn.

Maybe this complexity dissimulated into a simple outfit is the reason why this game is so addictive for grownups.

In 1972 Newell and Simon even made a computer play X and O and identified several strategies for winning and avoid the draw.

More information and a game example

Project Math-GAMES YouTube-Channel:

https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Tic-Tac-Toe Game instruction (video on youtube):

https://www.youtube.com/watch?v=Zwn9v_VNXwo

https://www.youtube.com/watch?v=fqAQuinb9Mk

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 9

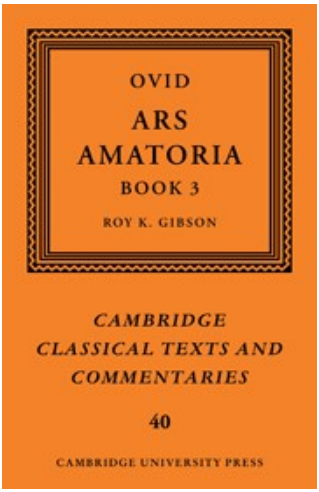
Geometry

- Line
- Right angle
- Square, rectangle
- Construct a 3x3-grid

Remark

We have to add that except the mathematical and strategy content, there is also an ethic content to be learned while playing this game. Maybe the most important thing to be considered in playing this game is fair play and modesty. A real master of this game is the one capable to identify and take full advantage of the other player's mistakes because there is no such thing as an unbeatable player and because you can always learn something new.

History



According to some researchers, the beginnings of the tic-tac-toe game can be traced back to the beginning of the first century BC in the Roman Empire. In his Third book of *Ars Amatoria*, Ovid speaks about a game called Terni Lapilli. The game was really popular and draws of its grid have been found chalked all over ancient Rome. Other researchers like Claudia Zaslavsky argue that Tic-tac-toe could originate back to ancient Egypt. In 1300 the game was extremely popular in England. In its simplest form the ancient game was very similar to the game that we play today. It was played on a 9 squares grid by two players. Each one had three stones in a distinct colour. The players had to put the stones on the grid one by one until all were placed. The winner was the one that managed to place his stones either in a horizontal, vertical or a diagonal roll. At this point the game ends and the players would start a new one. If all the stones

were placed on the grid but not in a straight line, a vertical or a diagonal one, the game was not over yet. It continued with the player moving the stones on the grid up, down, right or left, no diagonal move allowed, until a straight line was made. Many years later, in 1952, under the name OXO or Noughts and Crosses, Tic-Tac-Toe becomes one of the first video games. A computer was set to play perfect Tic-Tac-Toe against a human player.

Synonyms

Tic-Tac-Toe is also known as Noughts and Crosses or X's and O's.

In French the game is known under the name Tic-Tac-Toe or "morpion" – not to be confused with the "Morpion" game which is not very different but which requires to make lines of five and not of three.

Variants

- Over the time many variants were made.
- The simplest thing to do was to enlarge the grid.
- One of such variants is "Order and Chaos" which is played on a 6x6 grid. The first player is Order and the second is Chaos. Both control the X's and the O's. The first one, Order, has to make a line of five identical symbols while the Chaos has to prevent such a feat.

Interesting

In the "War Games film" (1983), the Tic-Tac-Toe game is used to teach a destructive computer that there are games one cannot win all the time.

References and Links

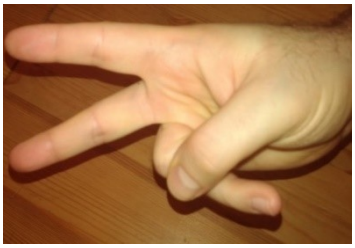
Wikipedia: https://en.wikipedia.org/wiki/Tic-tac-toe

Play Tic-Tac-Toe with the computer: http://playtictactoe.org/

4.3 Rock-Paper-Scissors

Overview

- **Genre:** hand game or ken game, simultaneous choice
- **Players:** 2
- **Age range:** older than 5
- **Setup Time:** instant
- **Playing Time:** instant
- **Random Chance:** yes



Game rules

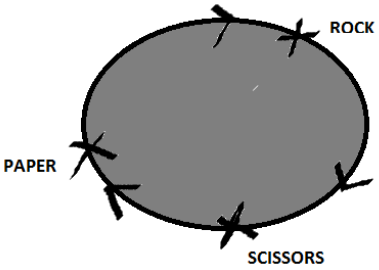
General

Rock-paper-scissors is a very simple but at the same time a very interesting game of simultaneous choice. It is played by two players who have to choose at the same time between three objects: rock (stone), paper (leaf in French) and scissors.

As you can imagine, it is possible to always have handy real paper, a rock and a pair of scissors. Each object out of these three is symbolized by a hand sign as shown in the picture above.

The winner is decided according to the following rule: the player who chooses the rock wins if his opponent chooses the scissors but loses if his opponent chooses the paper. If both players throw the same shape, the game is tied and is usually immediately replayed to break the tie. You may think that there is no logic in this rule but actually it is quite intuitive: the rock is superior to the scissors because it can smash them; the scissors are

superior to the paper because it can cut it and the paper is superior to the rock because it can cover it.



The rules can be summarized as follows:

- rock crushes scissors
- scissors cut paper
- paper covers rock

THE GAME PROCEEDS AS FOLLOWS:

- The two opponents outstretch their tight-fisted hands
- The two say in the same time rock, paper and do the sign of their choice
- If there is a draw, they will play again
- Sometimes the players decide to play a certain number of rounds and the one who wins more rounds wins the game.

It is now obvious that Rock-paper-scissors is a zero-sum game. In the table a win is noted with 1, a loss with -1 and a draw 0.

		PLAYER A		
		Rock	Scissors	Paper
PLAYER B	Rock	0	1	-1
	Scissors	-1	0	1
	Paper	1	-1	0

STRATEGY

Because Rock-paper-scissors is a zero-sum game, with incomplete information and based on hazard we can think that there is no strategy. This is not the case.

Over time mathematicians studied this game and tried to come up with solutions and strategies.

The game theory gives us the first solution. As we stand a chance out of three to win in order to maximize the odds we have to make a completely random choice. The problem and at the same time the potential advantage is that if a coin or a dice is not involved, humans are unable to make a totally random choice. Aristotle was completely right that man is a rational animal and the game theoreticians proved it 1000 years later, but that is another story. For this reason, behind every choice lies some strategy like in poker. So the player will constantly try to guess his opponent's strategy and take advantage of his weakness. This is why the game is actually intensely psychological. However, the experts consider that the best Rock-paper-scissors player is the one who manages to make his choices as random as possible.

References and Links

Project Math-GAMES YouTube-Channel: https://youtu.be/Tjf_Om75jQo

Wikipedia: https://en.wikipedia.org/wiki/Rock-paper-scissors

http://briselame.blogspot.be/2012/07/chifoumi.html

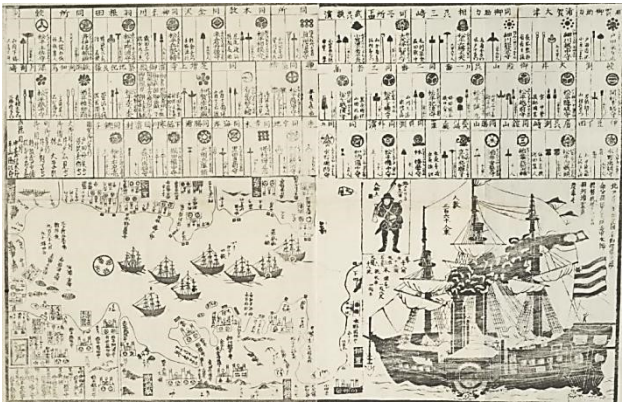
Rock-paper-scissor Game instruction (video on youtube):

https://www.youtube.com/watch?v=AnRYS02tvRA

What Mathematical Content can be Learned?

- This game is a strange case. There is in fact no math to be learned by playing this game but there is math to be learnt from it.
- The game has two aspects: the random one and the human one. So it can be used not just to exemplify the hazard theories but also for the study of the rational choice theories. It is the perfect study case for the game theory students.
- The mathematicians were also interested by it and tried to come up with algorithms to predict the moves and ensure victory, for instance by using the Marcov chain. Until now no definitive answer was found. The last experiment was made by researchers at the Tokyo U. They created a computer capable to predict the moves not by using an algorithm but an ultra-high speed camera which can predict the physical move of the player if not the strategy.

History



Right: Japanese representation from 1854 of Matthew Perry's "black ships"

Rock-paper-scissors finds its roots in Asia and more precisely in China. The game is firstly mentioned in a book from the Ming dynasty period which stated that the game is known since the time of the Han dynasty. In the 17th century the game was imported in Japan were it become very popular. Over time the Japanese version transformed a little.

In the 19th century, following the opening of Japan the game was exported to the West. In proof that the West adopted the Japanese variant stands the fact that the English name of the game is the literal translation of the Japanese name for the three hand-gestures. The Japanese variant is the only one in which the palm stands for paper. In the other Asian variants it stands for cloth.

In 1927 the game was already popular in England and France. In the same year a children's magazine in France described it in detail, referring to it as a "jeu japonais" ("Japanese game"). Its French name, "Chi-fou-mi", is based on the Old Japanese words for "one, two, three" ("hi, fu, mi"). In 1932 the game is presented in an article in *The New York Times* and in 1933 it is introduced in the *Compton's Pictured Encyclopaedia*.

Synonyms and Variants

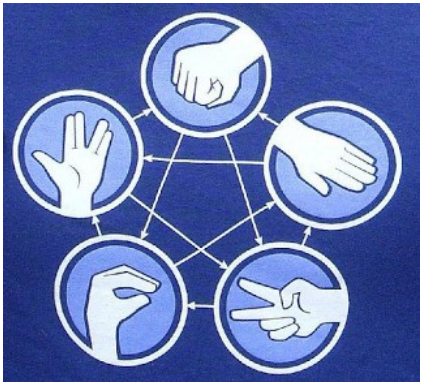
The game knows a series of regional names : papier-caillou-ciseaux, chifoumi or shifumi in France, pierre-papier-ciseaux-Baguette Magique or pierre-papier-ciseaux-puits in France and Belgium, roche-papier-ciseaux in Québec and New Brunswick, feuille-caillou-ciseaux or papier-marteau-ciseaux in Switzerland, Schnick-Schnack-Schnuck in Germany.

The easiest way to make a new variant is to add a new element. The first element to be added was the well. The result was really interesting. By using the Nash

equilibrium from the game theory the best strategy was to never use the rock and play as in the three elements game. In general if the game has an even number of symbols it will be really unbalanced: some symbols are more powerful than others. For this reason a 5th element was added: the bomb for example.

One of the most popular 5 elements variant is the one which was presented in "the big bang theory" film: rock, paper, scissors, lizard and Spock (a fictive person):

- The scissors cut the paper
- The paper covers the rock
- The rock smashes the lizard
- The lizard poisons Spock
- Spock breaks the scissors
- The scissors cut the head of the lizard
- The lizard eats the paper
- The paper disclaims Spock
- Spock vaporizes the rock
- The rock breaks the scissors



Interesting Facts

In 2005 Takashi Hashiyama, CEO of the Japanese television equipment manufacturer Maspro Denkoh, decided to sell the company's collection of impressionist art. To do so, he contacted the two biggest auction houses: Christie's International and Sotheby's Holdings. The two came with very similar proposals and Takashi Hashiyama could not decide between the two and he didn't want to split the collection. He asked the auction houses to find a solution between the two of them but this didn't succeed. In the end he asked them to play Rock-Paper-Scissors. He explained his decision by saying that "it probably looks strange to others, but I believe this is the best way to decide between two things which are equally good". The two houses had a week to decide the move. Christie's went to the 11-year-old twin daughters of the international director of Christie's Impressionist and Modern Art Department Nicholas Maclean, who suggested "scissors" because "Everybody expects you to choose 'rock'." Sotheby's Holdings decided to play "paper" without any strategy they said. Finally Christie's won the game and millions of dollars of commission.

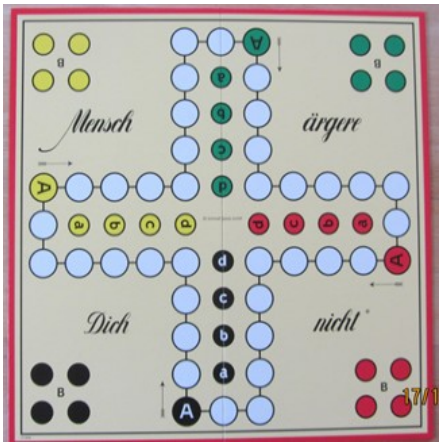
5.1 MENSCH ÄRGERE DICH NICHT! (LUDO)

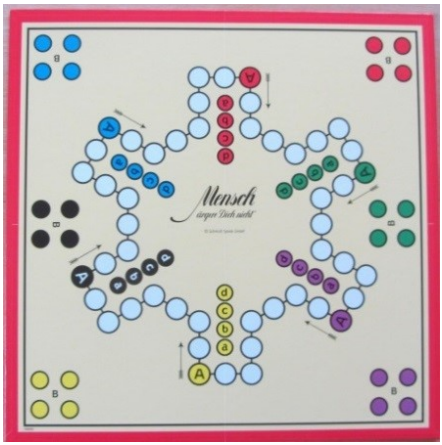
The game was invented by Josef Friedrich Schmidt 1907/1908 in Munich. “Ludo” is in German “Mensch ärgere Dich nicht”. It is considered as the most popular traditional board game in Germany.

OVERVIEW

Genre: strategy board game

- **Players:** 2 to 4 (2 to 6 on reverse side)
- **Age range:** 5 years and up
- **Setup Time:** 1 minute
- **Playing Time:** about 30 minutes
- **Random Chance:** High (dice rolling)
- **Skill(s) required :** Counting, probability, strategy, statistics





GAME RULES

GENERAL

“Mensch ärgere dich nicht” (Ludo) is a board game for two to four players, in which the players race their four figures from start to finish according to dice rolls. Like other cross and circle games, Ludo is derived from the Indian game Pachisi, but simpler. The game and its variants are popular in many countries and under various names. The game encourages logical, mathematical and strategic thinking. In addition you can learn positive discipline!

“Mensch ärgere dich nicht“ (engl. „Man, don’t get angry“, LUDO) is gender neutral. The name of the game derives from the fact that under certain conditions the play figures of other players are sent back to the waiting field. This is irritating for the affected player.

The game can be played by 2, 3 or 4 players – one player per board side (the original board has a pattern for 6 players on the reverse). Each player has 4 game pieces (figures), which are in the "out" area when the game starts and which must be brought into the player's "home" row. Early games had painted wooden pieces. The figures are placed in the “out”-area (marked “B”).

The home- rows are arranged in a cross pattern. They are surrounded and connected with a circle of fields over which the game pieces move in clockwise direction. There are 3 fields nearest to each side of the board; the left one is the player's "start" field (marked "A") and the middle one leads to the "home" row. (Marked with “a, b, c, d”)

This means that each game piece enters the circle at the "start" field, moves (clockwise) over the board and finally enters the "home" row. The first player with all of the pieces in the "home" row wins the game.

Starting the Game

The dice decides who will start. Each player throws the dice once. The player with the highest number starts the game. To be able to start now he needs a Six. For that he has to try three throws. If a Six is falling, it continues clockwise and his left neighbour may try his luck. If a Six is falling, the player can set his first figure on the start field. Now the hunt begins.

Game Progress

At least initially it is important to throw highest possible numbers. Without getting caught by the “enemy” the field must be circled once as quickly as possible. And the figure has to be brought into the field of “home”. Only the number, which was thrown, must be drawn. If a farther Six is thrown, the second figure is used, until all four figures are in the race.

Winner

The player who succeeds to bring home all four figures first is the winner. The remaining players continue to play (mostly). Now they play for the order of rank 2-4 (6).

Classical Rules of the Game

Three Times Rolling the Dice

Three times a player may throw the dice when he has no more figures on the field. The figures which are already standing in the home-field do not count.

Outplacing is Duty

Throwing a Six means: Bringing a piece into the game (by placing one from the "out" area (B) into the "start" field (A)) and throwing the dice again. Only if all four pieces are used in the game-field the Six could be used otherwise. The only exception: A player throws twice in succession the Six, so he cannot set free the starting field (A) for the next piece. In this case, he must use the Six for clearing the starting field.

In some variants a player who has no pieces in the game has three tries to throw a Six. If a piece is on the "start" field and there are still pieces in the "out" area, it must be moved as soon as possible. If a piece cannot be brought into the game – in this case any other piece in the game must be moved by the thrown number, if it is possible.

Pieces can jump over other pieces and throw out pieces from other players (into the player's "out" area) if they land on them. A player cannot throw out his own pieces though, and cannot advance further than the last field in the "home" row. A player cannot be thrown out, if he is on his "start" field.

Vacating the Starting Field

The starting –field must always be free of own pieces. This rule always has priority even if an “enemy” could be thrown out. The only exception: If no more pieces are in the own waiting field (B). In this case the starting field is losing his significance and other rules have priority.

Moving on Normally

The thrown number determinates the fields, which have to be set with one of the pieces in a clockwise direction. If a player has several pieces on the field, he has the choice, which piece he would like to replace. (Note the exceptions and obligations!) He moves on according to the thrown number. It doesn't matter if the fields are occupied by other pieces. If the goal-field is occupied by an "enemy"-piece, he throws it out and puts it back into the waiting area of the other player. If his goal-field is occupied by an own piece, then he has to move with one of his other figures.

Kicking Out is Duty

If his goal-field is occupied by an "enemy"-piece he is obliged to kick out this figure. Ludo! If a player notices an opportunity for throwing, it is allowed to remove the piece and to set it into the waiting area - the tables are turned!

More Information

Original game provider: http://www.schmidtspiele.de/so-viele-varianten.html

Instruction on Youtube in German: https://www.youtube.com/watch?v=FyLukIwR428

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 6 items
- Read and write numbers up to 40
- Order and compare numbers up to 6,
- Add single-digit numbers with totals to 6
- Subtract single-digit numbers from numbers up to 6
- coordinate different figures in their positions (holistic overview)
- Compare, count and organize own and other pieces

PROBABILITY CALCULATION AND STATISTICS

- Roll the dice and estimate the probability
- Various statistics of the game and the players.

SYNONYMS

The plagiarisms have never reached the success of the original. Even if the idea of the game doesn't know any borders. In Switzerland players are annoyed at "Eile mit Weile". The French roll the dice with „T'en fais pas". The Italians are asking „Non tarrabiare". In Spain the German classic game has the short name „Parchis". The Americans call it "frustration" and the English named it "Ludo". In Netherlands the game is called „Men's eager rivet".

VARIANTS

Online Express: In the shortened version of the classic you have to dodge different obstacles and opponents. These don't hesitate to throw you out. Get to know the popular game in a new way and prove tactical skill to bring your pieces into the home area with only a few moves.

Mauerhüpfer: In the variant "Hopping over the wall" you can easily choose the long way or the abbreviations over the wall, because it is played with two dice. These must be used in a clever way to save pieces. If young or old - the popular original "Man, don't get angry" also known as "MäDn" has already brought annoying for generations . And what annoys one gives the other pleasure!

HISTORY

At the beginning the game became known in the First World War. The game was invented by Josef Friedrich Schmidt 1907/1908 in Munich. He knew the ancient Indian running game

"Pachisi" and probably the British development "Ludo". For the first time it was sold in 1910 only with little success. With the beginning of the First World War the game became known by a marketing gag: Schmidt sent 3000 copies of his game to the battlefront, especially to war hospitals - so German soldiers were distracted from their suffering. After the war the soldiers took it home. The game became quickly popular in the civil society. In 1920 already one million games were sold. Today “Mensch ärgere dich nicht” is considered as the most popular traditional board game in Germany.

References and Links

http://www.schmidtspiele.de/produkt-detail/product/mensch-aergere-dich-nicht-mauerhuepfer-49276.html

Wikipedia: https://en.wikipedia.org/wiki/Mensch_%C3%A4rgere_dich_nicht

Sebastian Wenzel, Spiegel online, 29.1.2014 "Mensch ärgere Dich nicht" Spieleabend im Schützengraben
http://www.spiegel.de/einestages/mensch-aergere-dich-nicht-der-brettspiel-klassiker-wird-100-a-953259.html

Johann Osel: Bühne für Schadenfreude, 16.03.2014 in der Süddeutschen Zeitung
http://www.sueddeutsche.de/leben/jahre-mensch-aergere-dich-nicht-buehne-fuer-schadenfreude-1.1912772

Probability
http://www.mathe-online.at/lernpfade/KombinatorikundWahrscheinlichkeit/?kapitel=2
https://www.youtube.com/watch?v=6orKRQEPpqQ

Rules and variations
http://spielfibel.de/mensch-aerger-dich-nicht-spielregeln.php
http://www.schmidtspiele.de/produkt-detail/product/mensch-aergere-dich-nicht-mauerhuepfer-49276.html
http://onlinespiele.schmidtspiele.de/game/maednsp/platform/schmidt/

5.2 Seven Steps (Siebenschritt)

"Siebenschritt" is a traditional dance in Austria and Bavaria (Northern parts of the Alps), for any age, old and young people. The "Seven Steps" is danced in every day clothes or in traditional clothes.

Overview

- **Genre:** Traditional dance
- **Dancers:** couples in a line or circle in a room or in an outdoor space
- **Age range:** older than six years
- **Setup Time:** less than 1 minute (playing life music or recorded music or singing in the group)
- **Playing Time:** 3 minutes minimum (about 10 x the basic melody)
- **Random Chance:** none

Game rules / Dance Description

General

The dance is performed in a group with couples. Young and old, men and women, boys and girls can form a couple. The couples are dancing in a circle or in a line. This is a basic form, quite simple to learn, and many variants which are a little bit more complicated.

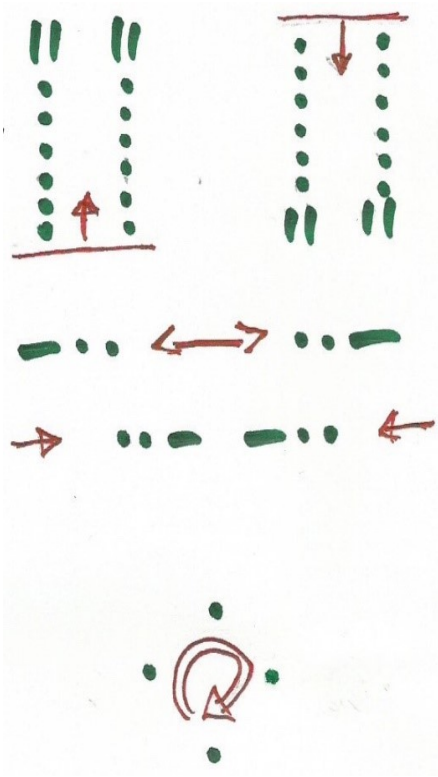
The dance has three phases. The photos show the beginning of each phase.

Phase a: steps forward, phase b): steps outside, phase c): steps in a circle

a) Beginning of the first phase: Seven steps forward. Both partners are beginning with the outside foot: the woman with her right foot and the man with his left foot.

b) At the beginning of the second phase: Stepping outside, hands are detaching. So these steps are performed with detached hands.

c) The third phase: Starting the circle to the right.



THE DANCE PROCEEDS IN THREE PHASES:

- Stepping with seven steps forward and backward
- Stepping with three steps outside and three steps inside
- Stepping with four steps in a circle

1st Phase: Seven steps forward and backward

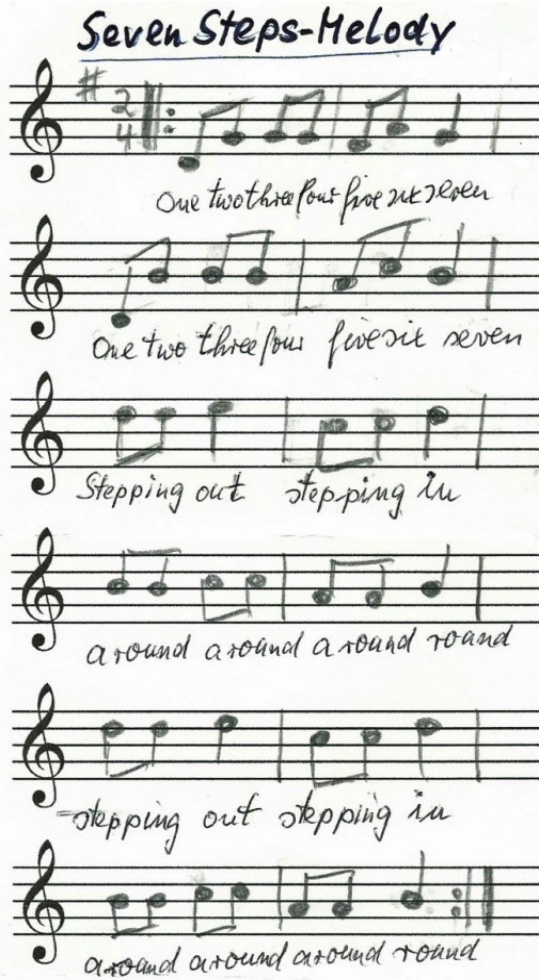
Both partners are holding hands. The hands at the outside may be on the hip or held up in the air. Both partners are beginning with the outside foot: the woman with her right foot and the man with his left foot.

The rhythm follows the text of the melody: six times “short” and then at the end one time “long”: short — short — short — short — short — short — long. On the graphic it is designed as . . . . . -

Stepping backward with seven steps follows the same pattern.

2nd Phase: Three steps outside and three steps inside

Both partners let go of the hand. They can place their hand on the hip. Both partners begin to step outward with the outside foot: the woman with her right foot and the man with his left foot. The rhythm follows the text of the melody: two times “short” and one time “long”: short — short — long. On the graphic it is designed as . . -
Stepping inward with three steps follows the same pattern.



3rd Phase: Four steps in a circle

Each couple starts at the beginning of this phase in a normal dance position and is performing the circle to the

right side with four steps. The partner on the left side is beginning with the left foot, and the partner on the right side is beginning with the right foot. The pattern is: long — long — long — long as signed in the graphic.

Strategy

There are different approaches to learn dancing. The main approaches are the following:

- Intuitive learning by imitation first and then the training of the exact steps and the structure of the dance.
- Learning the exact steps and the structure of the dance first and after that developing the feeling for the rhythm.

There are different combinations of these two types. For example: you may listen to the music first and get a feeling for the rhythm and the melody. You can even encourage the learners to sing by themselves. Then the teacher may demonstrate the dancing steps. Another alternative: the learners may accompany the trainer by clapping their hands.

More information and dance examples

Project Math GAMES YouTube-Channel:

- Seven Steps in Murnau/Bavaria: https://www.youtube.com/watch?v=owTuPHR-gVA
- A test of the dance in Messini /Greece: https://www.youtube.com/watch?v=I16t9fup14Q

Notes in two voices and videos:
http://www.volksmusik.cc/volkstanz/siebenschritt.htm

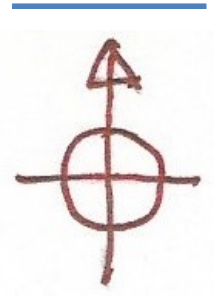
Some more videos as examples:

- A tutorial in German and the dance with music: https://www.youtube.com/watch?v=aMpreYhSA1I
- With variants of steps and music performed on a stage: https://www.youtube.com/watch?v=Ck-P8VBzdhs

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 7 items
- **Identify** different counting systems: footsteps on the ground (up to 7 items), tact of the melody (up to 4 or 8 items: 4 quarter notes and 8 eighth notes), rhythm (quarter notes and eighth notes), including zero (as a symbol for a break)
- **Identify** different combinations of these different counting systems



Geometry

- Line
- Circle
- Cross / crossing lines

History

Seven days of a week, seven colours of the spectrum, and seven notes of the musical scale - the number seven has a special meaning in everyday life, and in many cultures and religions. For Pythagoras, ancient Greek philosopher and mathematician, the seven is the combination of four and three. Four is standing for the earth with its four elements and three is a spiritual number.

Based on these traditions the dance Seven Steps is performed in a combination of 7 — 3 — 4 steps. The first documented melody of the dance “Siebenschritt” was written in Germany by Samual Scheidt in the 16th century. The melody was well known in England in medieval times as well, but then it was forgotten in England. Nowadays the dance Seven Steps / Siebenschritt is quite common in the Northern parts of the Alps, especially in Bavaria and Austria.

These are lots of different words for this melody, e.g. a song for children well known all over Germany: “Brüderchen komm tanz mit mir, meine Hände reich ich dir” (Little brother dance with me, I am giving you my hands). This song is part of the opera “Hänsel und Gretel” by E. Humperdinck.

Variants

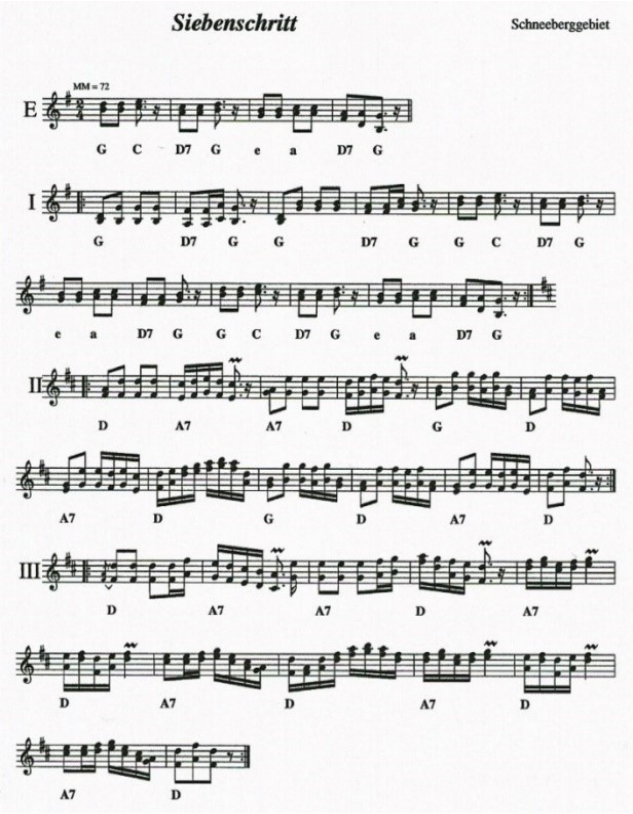
There are many variants according to local and regional traditions as can be seen on the videos on youtube. Two selected examples concerning, steps and holding hands:

Melody in two voices

See the notes at the right side.

Both hands on the back

Holding both hands on the back (phase 1 of the dance).



Dancing the circle

Dancing the circle (phase 3) by turning around in 4 hopping steps.

References and Links

Notes in two voices: http://www.volksmusik.cc/volkstanz/siebenschritt.htm

Music: http://videos.dancilla.com/m/s/at/arge/bgld/001/Clip23.mp3

Saptapadi (English: seven steps) is the most important rite of a Hindu marriage ceremony. See Wikipedia: https://en.wikipedia.org/wiki/Saptapadi

Song “Brüderchen komm tanz‘ mit mir” (Opera “Hänsel und Gretel” by Humperdinck): https://www.youtube.com/watch?v=baw1Y1GHsgU

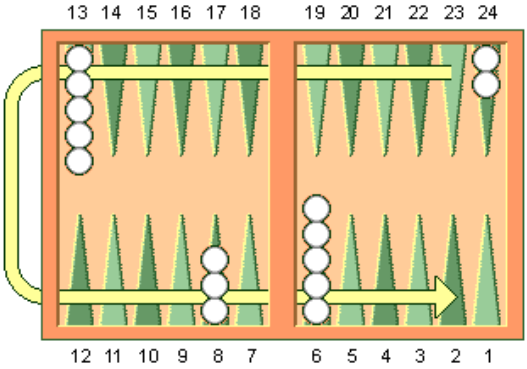
6.1 BACKGAMMON

Backgammon is one of the oldest board games known. It is a game for two players where playing pieces are moved according to the roll of dice, and a player wins by removing all of his pieces from the board. Backgammon is a member of the tables' family, one of the oldest classes of board games in the world.

OVERVIEW

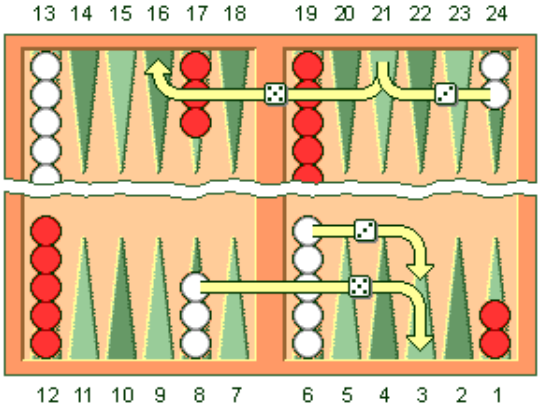
- Genre: strategy board game
- Players: 2, each has 15 pieces
- Age range: older than 5
- Setup Time: less than 1 minute
- Playing time: 5-30 minutes
- Random Chance: Medium (dice rolling)

GAME RULES



There are many different games one can play on a backgammon board, each with its own set of backgammon rules and different starting positions, but "standard backgammon" is the international version played at

live backgammon tournaments worldwide, and on most Internet backgammon play sites.



Backgammon is a game for two players, played on a board consisting of twenty-four narrow triangles called points. The triangles alternate in colour and are grouped into four quadrants of six triangles each. The quadrants are referred to as a player's home board and outer board, and the opponent's home board and outer board. The home and outer boards are separated from each other by a ridge down the centre of the board called the bar. Each player takes 15 playing pieces (checkers) of the same colour.

Object of the Game

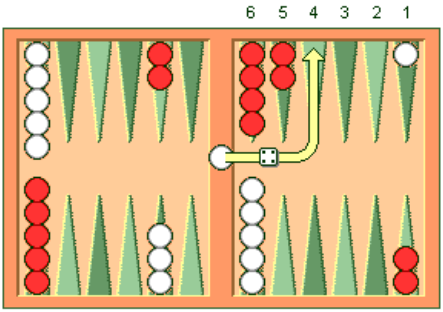
The object of the game is move all your checkers into your own home board and then bear them off. The first player to bear off all of their checkers wins the game.

Optional Rules

The following optional rules are in widespread use:

- Automatic doubles
  If identical numbers are thrown on the first roll, the stakes are doubled. The doubling cube is turned to 2 and remains in the middle. Players usually agree to limit the number of automatic doubles to one per game.
- Beavers
  When a player is doubled, he may immediately redouble (beaver) while retaining possession of the cube. The original doubler has the option of accepting or refusing as with a normal double.
- The Jacoby Rule
  Gammons and backgammons count only as a single game if neither player has offered a double during the course of the game. This rule speeds up play by eliminating situations where a player avoids doubling so he can play on for a gammon.

Starting a Game - Movement of the Checkers



To start the game, each player throws a single die. This determines both the player to go first and the numbers to be played. If equal numbers come up, then both players roll again until they roll different numbers. The player throwing the higher number now moves his checkers according to the numbers showing on both dice. After the first roll, the players throw two dice and alternate turns. The roll of the dice indicates how many points, or pips, the player is to move his checkers. The checkers are always moved forward, to a lower-numbered point. The following rules apply:

A checker may be moved only to an open point, one that is not occupied by two or more opposing checkers.

The numbers on the two dice constitute separate moves. For example, if a player rolls 5 and 3, he may move one checker five spaces to an open point and another checker three spaces to an open point, or he may move the one checker a total of eight spaces to an open point, but only if the intermediate point (either three or five spaces from the starting point) is also open.

A player who rolls doubles plays the numbers shown on the dice twice. A roll of 6 and 6 means that the player has four sixes to use, and he may move any combination of checkers he feels appropriate to complete this requirement. A player must use both numbers of a roll if this is legally possible (and all four numbers of a double). When only one number can be played, the player must play that number. Or if either number can be played but not both, the player must play the larger one. When neither number can be used, the player loses his turn. In the case of doubles, when all four numbers cannot be played, the player must play as many numbers as he can.

Hitting and Entering

A point occupied by a single checker of either colour is called a blot. If an opposing checker lands on a blot, the blot is hit and placed on the bar.

Any time a player has one or more checkers on the bar, his first obligation is to enter those checker(s) into the opposing home board. A checker is entered by moving it to an open point corresponding to one of the numbers on the rolled dice.

For example, if a player rolls 4 and 6, he may enter a checker onto either the opponent's four point or six points, so long as the prospective point is not occupied by two or more of the opponent's checkers.

If neither of the points is open, the player loses his turn. If a player is able to enter some but not all of his checkers, he must enter as many as he can and then forfeit the remainder of his turn.

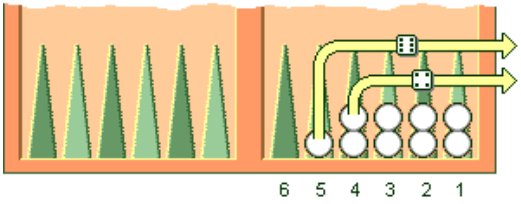
After the last of a player's checkers has been entered, any unused numbers on the dice must be played, by moving either the checker that was entered or a different checker.

Bearing Off

Once a player has moved all of his fifteen checkers into his home board, he may commence bearing off. A player bears off a checker by rolling a number that corresponds to the point on which the checker resides, and then removing that checker from the board. Thus, rolling a 6 permits the player to remove a checker from the six points.

If there is no checker on the point indicated by the roll, the player must make a legal move using a checker on a higher-numbered point. If there are no checkers on higher-numbered points, the player is permitted (and required) to remove a checker from the highest point on which one of his checkers resides. A player is under no obligation to bear off if he can make an otherwise legal move.

White rolls  and bears off two checkers.



A player must have all of his active checkers in his home board in order to bear off. If a checker is hit during the bear-off process, the player must bring that checker back to his home board before continuing to bear off. The first player to bear off all fifteen checkers wins the game.

Scoring

The first player to bear off all his checkers wins the game. However, if an opponent fails to bear off at least one checker from his home board or is caught with one or more checkers still outside his home board, in the outer board areas, the winner scores a “gammon”. A gammon is worth twice the points or stakes being played for in “match play”, a series of games played up to a certain number of points, or, twice the current wager in a “money game”, a series of single games played at predefined stakes.

More information and a game example

Project Math-GAMES YouTube-Channel:

https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

Backgammon Game instruction in English (video on youtube):

https://www.youtube.com/watch?v=v9yKQ8QeIOY

What Mathematical Content can be Learned?

Numeracy

- Coordinate system
- Count reliably up to 36
- Subtraction with numbers up to 15
- Calculate with single-digit numbers
- Read and write numbers up to 9, including zero
- Order numbers up to 15, Compare numbers
- Identify the range of possible outcome when using more dices

Geometry

- Recognize and name two dimensional shapes (square, rectangle, triangle)
- Recognize and name the three dimensional shape cube

HISTORY

The oldest and greatest of games, Backgammon has a long and very intriguing history - it has been known by many different names and variants for more than 5000 years!

Archaeological evidence unearthed in the year 2004 - specifically the oldest known board, made of ebony with playing pieces of agate and turquoise stone - shows that a variant of the game was first played about 5200 years ago in the legendary Burnt City of the Sistani-Baluchistan province in south-eastern Iran.

Variants of backgammon were played later by many early cultures, in China, India, Egypt, Greece and Rome from where it spread to numerous countries to enjoy eras of immense popularity and earn the title of "The King of Games". In some societies it could only be played by the upper class, aristocracy and royalty and thus is also known as "The Game of Kings".

Later backgammon reached Europe, to England, France, Spain, Italy and Holland. The modern game of backgammon is alleged to have stemmed from a version called tables played in 17th century England, one that evolved into a game where doublets were played twice and one would win twice or triple the stakes when an opponent would fail to remove or get home any of his checkers.

Certain passages in literature indicate the name backgammon was first used around the mid-1600s although the exact origin of the word is not clear. It may have come from the Welsh words beak (back) and camion (battle) or possibly the Middle English words of beak (back) and gamin (game).

The famous writer of games, Edmond

Hoyle, published a treatise on backgammon in 1745 with a set of rules, and even some strategy tips, that still ring true today.

Then in the 1920s, the invention of the doubling cube by an unknown player in New York City upped the stakes and sparked new interest and excitement to the game. Backgammon became quite popular in that era and in 1931 the New York Racquet and Tennis Club's backgammon committee, headed by Wheaton Vaughan, wrote a set of backgammon rules that are the actual source of the standard rules used internationally today.

From the 1960s through to 1980s, the game was a huge fad and backgammon boards were a fixture at most clubs and discothèques in North America. Many big live tournaments were organized, including the first World Championship, and people young and old, from all walks of life, were playing the game everywhere night and day.

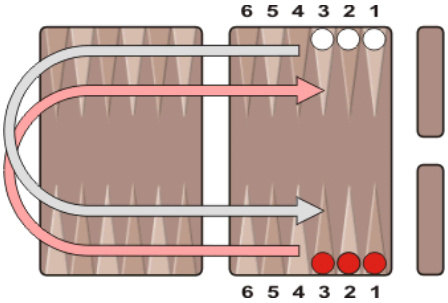
Variants

There are many variants of standard backgammon rules. Some are played primarily throughout one geographic region, and others add new tactical elements to the game. Variants commonly alter the starting position, restrict certain moves, or assign special value to certain dice rolls, but in some geographic regions even the rules and directions of the checkers movement change, rendering the game fundamentally different.

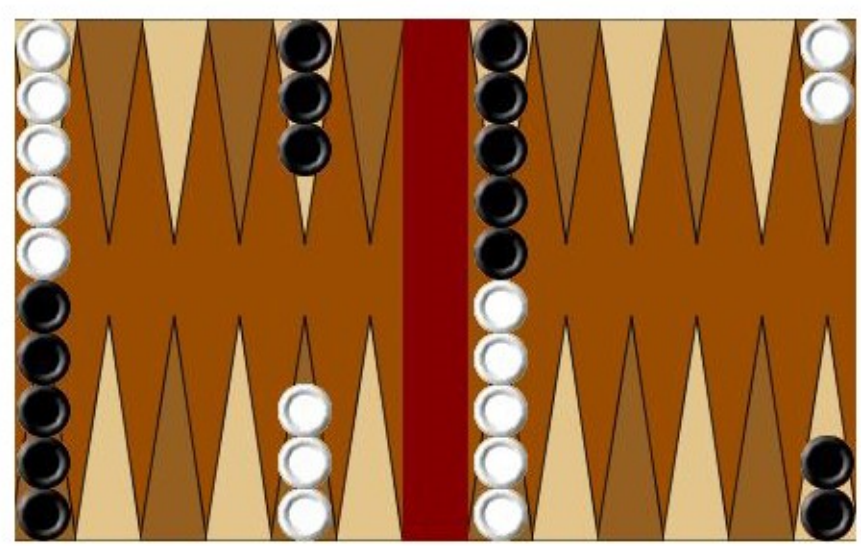
Nack-Gammon

Nack-Gammon is a version of the game invented by Nack Ballard, the number-1 Giant of Backgammon. It is designed to add more skill and creativity to the game. It also uses the same rules as standard backgammon – the difference is the starting position as shown below – with two checkers on the 23 points, one taken each from the 13 and 8 points, where normally five checkers go.

HYPER-GAMMON



Hyper-Gammon is a fast-paced variant that is played with only three checkers per side using the same rules as in standard backgammon. The starting position is shown in the image below. You are playing Red and you begin with one checker on each of your three furthest points. Everything else is the same – you can play for fun or for money using the cube and the Jacoby Rule.



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http://www.bkgm.com/rules.html

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http://www.gammonlife.com

Wikipedia:

https://en.wikipedia.org/wiki/Backgammon

6.2 Chess

Chess is a two-player board game played on a chessboard, a checkered game board with 64 squares arranged in an eight-by-eight grid. Chess is played by millions of people worldwide in homes, urban parks, clubs, online, correspondence and in tournaments.

Overview

- Genre: strategy board game
- Players: 2, each has 16 pieces
- Age range: older than 5
- Setup Time: less than 1 minute
- Random Chance: none, no dice needed

Game rules

Chess is a game played between two opponents on opposite sides of a board containing 64 squares of alternating colours. Each player has 16 pieces: 1 king, 1 queen, 2 rooks, 2 bishops, 2 knights, and 8 pawns.

The objective of each player is to place the opponent's king "under attack" in such a way that the opponent has no legal move. The player who achieves this goal is said to have "checkmated" the opponent's king and to have won the game. Leaving one's own king under attack, exposing one's own king to attack and also "capturing" the opponent's king are not allowed. The opponent whose king has been checkmated has lost the game.

If the position is such that neither player can possibly checkmate, the game is drawn.

Starting a Game

At the beginning of the game the chessboard is laid out so that each player has the white (or light) colour square in the bottom right-hand side. The chess pieces are then arranged the same way each time. The second row (or rank) is filled with pawns. The rooks go in the corners, then the knights next to them, followed by the bishops, and finally the queen, who always goes on her own matching colour (white queen on white, black queen on black), and the king on the remaining square.

The player with the white pieces always moves first. Therefore, players generally decide who will get to be white by chance or luck such as flipping a coin or having one player guess the colour of the hidden pawn in the other player's hand. White then makes a move, followed by black, then white again, then black and so on until the end of the game.

How the Pieces Move

Each of the 6 different kinds of pieces moves differently. Pieces cannot move through other pieces (though the knight can jump over other pieces), and can never move onto a square with one of their own pieces. However, they can be moved to take the place of an opponent's piece which is then captured. Pieces are generally moved into positions where they can capture other pieces (by landing on their square and then replacing them), defend their own pieces in case of capture, or control important squares in the game.

The King

The king is the most important piece, but is one of the weakest. The king can only move one square in any direction - up, down, to the sides, and diagonally.

The Queen

The queen is the most powerful piece. She can move in any one straight direction - forward, backward, sideways, or diagonally - as far as possible as long as she does not move through any of her own pieces. And, like with all pieces, if the queen captures an opponent's piece her move is over.

The Rook

The rook may move as far as it wants, but only forward, backward, and to the sides. The rooks are particularly powerful pieces when they are protecting each other and working together!

The Bishop

The bishop may move as far as it wants, but only diagonally. Each bishop starts on one colour (light or dark) and must always stay on that colour. Bishops work well together because they cover up each other's weaknesses.

The Knight

Knights move in a very different way from the other pieces – going two squares in one direction, and then one more move at a 90 degree angle, just like the shape of an "L". Knights are also the only pieces that can move over other pieces.

The Pawn

Pawns are unusual figures, because they move and capture in different ways: they move forward, but capture diagonally. Pawns can only move forward one square at a time, except for their very first move where they can move forward two squares. Pawns can only capture one square diagonally in front of them. They can never move or capture backwards. If there is another piece directly in front of a pawn he cannot move past or capture that piece.

Promotion

Pawns have another special ability and that is that if a pawn reaches the other side of the board it can become any other chess piece (called promotion). A pawn may be promoted to any piece. A pawn is usually promoted to a queen. Only pawns may be promoted.

En Passant

The last rule about pawns is called “en passant”, which is French for “in passing”. If a pawn moves out two squares on its first move, and by doing so lands to the side of an opponent’s pawn (effectively jumping past the other pawn’s ability to capture it), that other pawn has the option of capturing the first pawn as it passes by. This special move must be done immediately after the first pawn has moved past; otherwise the option to capture it is no longer available.

More information and a game example

Chess Game instruction in English (video on youtube):

https://www.youtube.com/watch?v=cA-fA5eLGpY

What Mathematical Content can be Learned?

Numeracy

- Understand a coordinate system and find positions up to 8
- Count reliably up to 20 or 50
- By worth of chess pieces:
    - Addition of single-digit numbers with totals to 39
    - Subtraction of single-digit numbers from numbers up to 10
    - Equations (e.g. Knight = pawn + x)
- Read and write numbers up to 9, including zero
- Order and compare numbers up to 39, including zero

Geometry

- Recognize and name two dimensional shapes (square, rectangle, rhombus)
- Describe length and width of shapes
- Measure areas by counting squares

History

Iranian shatranj set, glazed fritware, 12th century. Metropolitan Museum of Art

The history of chess spans over 1500 years. Chess appeared in India around 600 A.D., was adopted in Persia around 700 A.D., and was absorbed by the Arab culture around 800 A.D. The Arab / Muslim influence was responsible for its later introduction into other cultures. The game of chess we know today has been around since the 15th century when it became popular in Europe. The "Romantic Era of Chess" was the predominant chess playing style down to the 1880s. It was characterized by swashbuckling attacks, clever combinations, brash piece sacrifices and dynamic games. Winning was secondary to winning with style. These games were focused more on artistic expression, rather than technical mastery or long-term planning. The Romantic era of play was followed by the Scientific, Hypermodern, and New Dynamism eras.

In the second half of the 19th century, modern chess tournament play began, and the first World Chess Championship was held in 1886. The 20th century saw great leaps forward in chess theory and the establishment of the World Chess Federation (FIDE). Developments in the 21st century include use of computers for analysis, which originated in the 1970s with the first programmed chess games on the market. Online gaming appeared in the mid-1990s.

Variants

Losing Chess

Losing chess (also known as Antichess, the Losing Game, Giveaway chess, Suicide chess, Killer chess, Must-Kill, or Take-all chess) is a chess variant in which the objective of each player is to lose all of his pieces or be stalemated, that is, a misère version. In some variations, a player may also win by checkmating or by being checkmated. Losing chess is one of the most popular of all chess variants.

The origin of the game is unknown, but believed to significantly predate an early version, named Take Me, played in the 1870s. Because of the popularity of losing chess, several variations have spawned.

Dunsany's chess

Dunsany's Chess, also known as Dunsany's game, is an asymmetric chess variant in which one side has standard chess pieces, and the other side has 32 pawns. Unlike many chess variants, this one does not feature any fairy pieces, which are pieces not found in conventional chess. This game was invented by Lord Dunsany in 1942. A similar game is called “horde chess”.

Chess960

Chess960 (or Fischer Random Chess) is a variant of chess invented and advocated by former World Chess Champion Bobby Fischer, publicly announced on June 19, 1996 in Buenos Aires, Argentina. It employs the same board and pieces as standard chess; however, the starting position of the pieces on the players' home ranks is randomized. The name "Chess960" is derived from the number of possible starting positions. The random setup renders the prospect of obtaining an advantage through the memorization of opening lines impracticable, compelling players to rely on their talent and creativity.

Randomizing the main pieces had long been known as Shuffle Chess; however, Chess960 introduces restrictions on the randomization, "preserving the dynamic nature of the game by retaining bishops of opposite colours for each player and the right to castle for both sides, resulting in 960 unique starting positions.

References and Links

https://www.fide.com/component/handbook/?id=124&view=article

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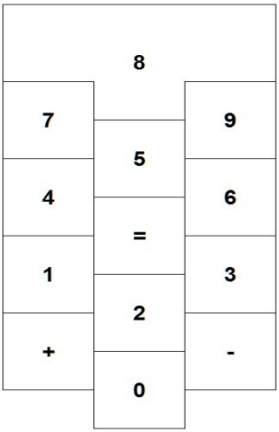
6.3 Calculator Hopscotch

Hopscotch is a game that has been around for over three hundred years. A favourite of children everywhere, you can play it with friends or all alone. The word hopscotch means "hop-scratch," since players sometimes scratch the court into dirt or slate on the ground, and then play by hopping over the scratched lines. Calculator-hopscotch is a mathematical variant of the original well known game.



Overview

- Genre: street game
- Players: 2 or more
- Age range: older than 6
- Setup Time: less than 1 minute
- Playing time: 5-60 minutes
- Random Chance: none

Game Rules

Traditionally, this game was carved with a stick in the dirt, but with the advent of sidewalk chalk, dirty feet are a thing of the past. A hopscotch course typically looks like a series of numbered squares. Draw a hopscotch design on the ground. Chalk is the best drawing medium on asphalt, patio stones or concrete. The squares should be large enough to fit one foot and to make sure that a stone thrown into the square will not bounce out too easily. While there are variants on drawing the design, a calculator hopscotch design and a "normal" math-hopscotch-design is shown here.

You have to solve as many equations as possible. Before the game the players decide the number of equations needed to win the game or they can set a time limit (e.g. 30 min.)

Starting a Game

There are three ways to play the game:

Solve it

The first player jumps from square to square to indicate an equation; for example, you might jump onto these in order: **4 + 3 =**. The next player jumps to the answer **7**. If the answer is a two-digit number, then you have to hop through the squares. If you succeed, you jump a different equation (the player after you has to solve that too, and so on). Each square gets one foot. Which foot you start with is up to you but you can't have more than one foot on the ground at a time. Always keep your feet inside the appropriate square(s); if you step on a line, hop on the wrong square, or step out of the square, you lose your turn. Your goal is to complete the course with the equations you agreed at the beginning. The first person to do this wins the game!

Time Challenge

On your turn, a player tosses a stone onto a number. Throw a flat stone or similar object (small beanbag, shell, button, plastic toy) to land on square one. It has to land inside the square without touching the border or bouncing out. If you don't get it within the lines, you lose your turn and pass the stone to the next person. If you do get it, however, go on to the next step.

You then see how many equations equalling that number you can jump in 60 seconds. Again, each square gets one foot. Always keep your feet inside the appropriate square. After one round when all the players have played, the one that found the most equations wins the game.

Co-op Calculations

In this variant of the game the players make equal numbered teams. The first team begins with one player tossing a stone on a number, then, taking turns, and

the others in the group try to jump as many different equations as possible equalling that number. The team with the highest number of different equations wins!

Game Example

Hopscotch Game instruction on YouTube Math-GAMES-channel:

https://www.youtube.com/watch?v=F81h01Asr7U

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 100
- Addition of single-digit numbers
- Subtraction with numbers up to 10
- Multiply using single-digit whole numbers
- Division with numbers up to 10
- Read and write numbers up to 9, including zero
- Order numbers up to 10
- Compare numbers

Geometry

- Recognize and name two dimensional shapes (square, rectangle)

References and Links

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http://www.grandparents.com/grandkids/activities-games-and-crafts/hopscotch

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http://www.wikihow.com/Play-Hopscotch

https://en.wikipedia.org/wiki/Hopscotch

7.1 Magic Square

Overview

- Genre: strategy logic game
- Players: 1
- Age range: older than 7
- Setup Time: less than 1 minute
- Playing Time: 3 minutes to 20 minutes
- Random Chance: none

Game rules

General

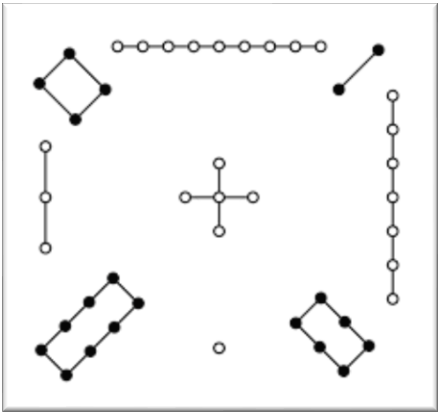
It is an excellent way to experience the fun of math while building both math and critical thinking skills. Magic squares have become very popular also thanks to the advent of mathematic games like Sudoku.

A magic square is an arrangement of numbers in a square in such a way that the sum of each row, column, and diagonal is one constant number, the so-called "magic constant."

A magic square consists of consecutive integers placed in a squared grid in which each row, column, and diagonal ads up to the same number, that we call Magic Number. The player has to draw all possible combinations inside the square so the numbers added together lead to the same number: the Magic Number!

A 1 by 1 magic square contains just the number 1 and is so simplistic, it is not worth discussing.

It is impossible to construct a 2 by 2 magic square (n = 2) and so the first magic square worth discussing occurs when n = 3



Odd Magic Squares

A 3 by 3 magic square is an odd magic square (n=3, 5, 7, 9, 11, etc), one of the three types of magic square.

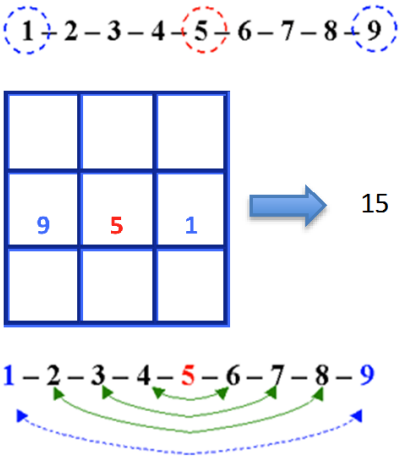
The other two types are

- Doubly even (multiple of 4 where n=4, 8, 12, 16, 20, etc.)
- Singly even (even but not a multiple of 4 where n=6, 10, 14, 18, 22, etc.)

How to Solve a 3x3 Magic Square?

1st phase:

Build a magic square of 3 x 3 = 9 boxes by using only once all the numbers from 1 to 9 and write them inside the square



2nd phase

Circle in blue the first (lower) and the last (higher) number and in red the central number

3rd phase

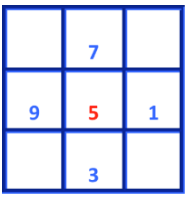
Complete a row, column or diagonal with the other two numbers, the greater and the lesser, that you have highlighted before and calculate the sum of three numbers. The magic constant (the sum of each row, column and diagonal) is, therefore, 15. Since number 5 is in the centre, the other remaining pairs of numbers whose sum is 10 must be arranged in the empty boxes of the rows, columns, and diagonals.

4TH PHASE

Write all pairs of numbers whose sum is 10 and arrange them in proper rows, columns and diagonals which are still empty.

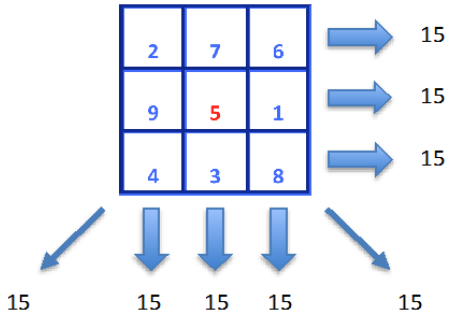
5TH PHASE

The remaining numbers are: 2, 3, 4, 6, 7, 8. Having put the 5 in the middle, pairs are: 8 + 2 , 3 + 7 , 4 + 6 . After some attempts you can find that the right pair to write in the central vertical row is 3 + 7



6TH PHASE

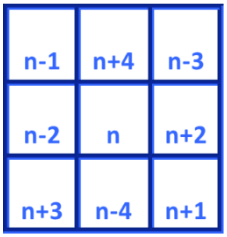
Only 4 numbers are missing, the couples 2 + 8 and 4 + 6. The two couples have to be inserting in the diagonals of the square. So you can magically understand that the numbers should be arranged in this way.



STRATEGY

One general solution for Magic Squares exists for any given total 3n (plus rotations and reflections of this). A key strategy is to identify the three sequential numbers that form one diagonal – that is, the centre number is the total of the square divided by three.

EXAMPLES, REFERENCES AND LINKS



https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g
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Which Mathematical Content can be Learned?

Numeracy

- Order and compare numbers up to 100, including zero
- Understand a coordinate system and find positions up to 100
- Add two-digit whole numbers
- Interpret +, - and = in practical situations for solving problems
- Use a calculator to check calculations using whole numbers
- Subtract single-digit numbers from numbers up to 100
- Add two-digit numbers with totals to 100
- Mental calculation
- Determine the approach, materials and strategies to be used

Geometry

- Know geometric lines (horizontal line, vertical line) and describe lines in two dimensional shapes (length, width, diagonal)
- Understand right angle
- Recognize and name two dimensional shapes (square, rectangle)
- Understand symmetry in shapes
- Understand concepts of diagonal, horizontal and longitudinal lines

Variants

4x4 magic squares

http://mathforum.org/alejandre/magic.square/adler/adler4.html

5x5 and other odd-numbered magic squares

http://mathforum.org/alejandre/magic.square/adler/adler.5x5math.html

History

Magic squares have a rich history dating back to around 2200 B.C. A Chinese book entitled Lo Shu, relates the Chinese myth about Emperor Yu who was walking along the Yellow River and noticed a tortoise with a diagram on its shell. He decided to call the unusual numerical pattern lo shu. The earliest magic square, however, appeared in the first-century book Da-Dai Liji. Magic squares in China

were used in different type of study, such as divination and astrology, philosophy, interpretation of natural phenomena and human behaviour.

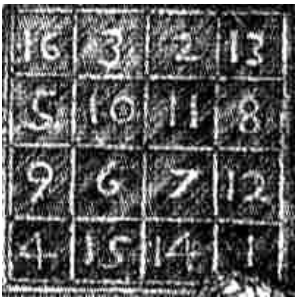
Magic squares travelled from China to India and to the Arab countries. Then they arrived in Europe and after in Japan. Magic squares in India were used for different purposes, also for the dissemination of mathematical knowledge. Treatises, between the 9th and 10th centuries, revealed that the mathematical properties of magic squares were already developed among Islamic Arabic nations. At that time the introduction of magic squares was entirely mathematical rather than magical. Later, during the 11th and 12th centuries, Islamic mathematicians proposed a series of simple rules to create magic squares. The magic and divination became associated to magic squares during the 13th when it seems that the Arabs produced the 10x10 magic square.

In West Africa the study of magic squares produced great interest too. In that period, in fact, the squares held particular spiritual importance and were inscribed on clothing, masks, and religious artifacts. They became even influential in the design and building of houses.

In 1300, the Byzantine writer Manuel Moschopoulos introduced magic squares in Europe with his treatise, where, as in other cultures, magic squares were linked with divination, alchemy, and astrology.

Magic squares also appeared in previous ages, in a Spanish manuscript written in 1280 actually set in the Biblioteca Vaticana by Alfonso X of Castille. In that text, each magic square is assigned to the respective planet, as in the Islamic literature. In the 14th century, Magic squares reappeared in Italy in Florence in a manuscript of the Trattato d'Abbaco (Treatise of the Abacus) written by Paolo Dagomari, a mathematician, astronomer and astrologer who was in contact with Jacopo Alighieri, son of Dante. The squares can be seen at the Biblioteca Universitaria of Bologna. They also appear in a manuscript copy of the Trattato dell'Abbaco from the 15th century.

In Europe, since the early 15th century, several manuscripts were produced about magic squares assigned to the seven planets, and described as a means to attract the influence of planets and their angels (or demons) during magical practices. In Cambridge we can find De liber de angelis, a magical handbook written around 1440. At the Biblioteka Jagiellonska, contained in the codex 793, there is “De septem quadraturis planetarum seu quadrati magici” another handbook of magic planetary images.



The first evidence of a magic square appearing in Europe was in 1514 the famous engraving Melancholia by the German artist Albrecht Dürer (4x4 square).

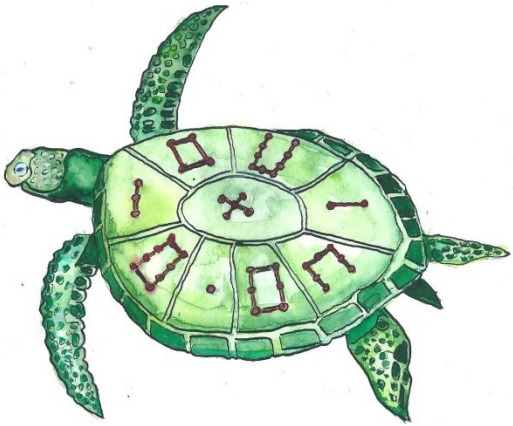
In 1592, Chen Dawei from China published his book Suan fa tong zog and launched the beginning of the study of magic squares in Japan. Magic squares became very popular and they were studied by many famous Wasan, who were Japanese mathematics experts. The oldest record of magic squares in Japanese history was evident in the book Kuchi-zusam, which described a 3-by-3 square.

The study of magic squares became very important during 17th century. In 1687-88, a French aristocrat, Antoine de la Loubere, studied the mathematical theory of constructing magic squares. In 1686, Adamas Kochansky, a Polish mathematician, extended magic squares to three dimensions. In the 18th century, Muhammad ibn Muhammad, a well-known astronomer, mathematician, mystic, and astrologer in Muslim West Africa, took an interest in magic squares and in one of his manuscripts; he gave examples of, and explained how to construct, odd-order magic squares.

Antoni Gaudi (1852 - 1926) inserted on a side of his Sagrada Familia Church in Barcelona, a magic square of order 4x4. The sum of its rows, columns and diagonals (and actually also of other "forms" internal squared) is always 3333, equal to the years of Christ at the time of the Passion.

During the latter part of the nineteenth century, mathematicians applied the squares to problems in probability and analysis. Today, magic squares are studied in relation to factor analysis, matrices, arithmetic, mathematics and geometry.

The oldest known Magic square, which is also the most common one, is called Lo-Shu. Here is the story:



The Emperor Version of the Legend Lo Shu

Standing at the river's edge, the Emperor Yu-Huang watched the Yellow River rush before him. The Emperor sought solitude because that he had had a very difficult day. Every day he dealt with taxes, an underpaid army and his angry wife. So, enjoying and looking out across the river, he could feel his problems slip away. One night, he walked to the river's edge alone. Looking out to the opposite side of the river, at that moment he saw the divine turtle. Emperor Yu had seen that turtle before, but never so close. The emperor knew that the turtle was a symbol of good luck and that it was the same turtle he had seen formed by a pattern of stars in the sky, each night before he went to bed. Wanting to get a better look at the magic turtle, the emperor took one step closer. The turtle didn't notice the emperor and continued to move slowly into the water. Therefore the Emperor could observe the detail of the shell, totally new to him. The turtle's hard back looks like puzzle pieces glued together to form two circles around a rectangle. Emperor Yu noticed nine symbols formed by a series of dots arranged in a square (hence the name Lo Shu). He discovered that this grid of nine sectors had a correspondence with the eight trigrams of the Pa Kua: the eight sectors of the octagon including the centre. These signs corresponded to nine numbers arranged so that added up in line horizontally, vertically or diagonally, the result always gave the number 15, which are the number of days that the moon takes to change from the old to the new one. Then, Emperor Yu established the correspondence between the Lo Shu, directions, and the eight trigrams, and saw that each of the numbers (except the five located in the centre) was addressed in one of eight directions or different solar energy. The emperor wondered about what all this meant. Was the divine turtle giving a signal? As the river became dark, the emperor decided to come back home. Walking slowly, he thought about the different numbers and their positions one to another. What did the magic sum have to do with him? Did it mean good luck? No answers could be found. The emperor was troubled. He had come to the river's edge to seek tranquillity, and instead had found doubt. However the Emperor solved the problem and was honoured as a hero. Everyone in China became very happy with the help of the Magic square on the divine turtle.

The Magic Square was remarkable because every horizontal, vertical, and diagonal row added up to 15. Fifteen is the number of days between the new moon and full moon respectively. The number five is highly regarded in ancient China and this magic square contained a number five in the centre.

Number in Lo-shu grid	Cardinal points	Colours	Elements	Significance	Explanation
1	North	White	Water	Communi-cation	How an individual reacts and communicates with others
2	South-west	Black	Earth	Sensitive & Intuition	Sensitive and intuition power
3	East	Blue	Wood	Planning & Imagination	Intellectual capability of an individual and memory and ability to clearly and logically think
4	South-east	Green	Wood	Discipline & Organization	How an individual is practical and hardworking. Also represents order and balance
5	Centre	Yellow	Earth	Balance (emotional & mental)	Emotional stability of an individual
6	North-west	White	Metal	Home & Family	Creativity and love for home and family. It relates an individual to be helpful with people and friendship.
7	West	Red	Metal	Disappoint-ments	Heart rules over head. The individual learns through losses and disappointments of love, emotion, health and finance, the spirit of sacrifice
8	North-east	White	Earth	Discipline & Organization	Attention to details
9	South	Purple	Fire	Humanitarian	Idealism, value and ambition

7.2 Four Seasons

"Four seasons" is a very old game. It can be played with a board

Overview

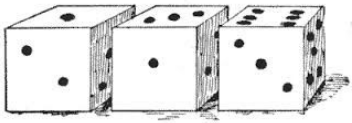
- **Genre**: strategy board game
- **Players**: 4, each has 12 pawns
- **Age range**: older than 5
- **Setup Time**: 1 minute
- **Playing Time**: half an hour or longer
- **Random Chance**: high in combination with strategy

Game Rules

GENERAL

Four seasons is a game for four players, played on a board consisting of four quadrants related to the four seasons. Each season has a different colour: green for spring, red for summer, black for autumn and white for winter. Each quadrant is referred as the player's home board. Each quadrant has narrow fields to put the pawns.

On the right you see a board with the pawns in their initial position.



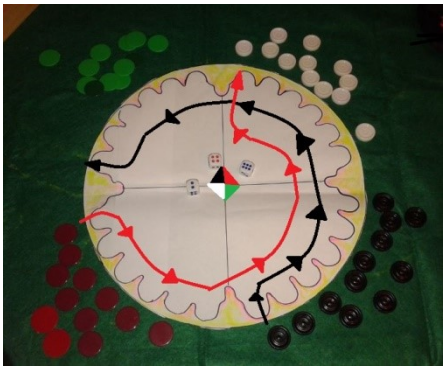
The coloured areas on the board represent the field to reach by each player to start the bearing off.

The point 1 represents the place where each player has to start the game. Each player has twelve pawns of his own colour. The initial arrangement of pawns is outside each quadrant. The players have three dice.

Objective of the Game

The objective of the game is to move all your pawns into your own home board and then to reach the opposite quadrant. The first player to bear off all the pawns wins the game. The game consists of three phases: Moving the pawns, hitting and entering pawns and bearing off.

1st Phase: Moving the Pawns



To start the game, each player throws a single die. This determines which the player goes first. If equal numbers come up, then the players roll again until they roll different numbers. The player throwing the higher number will start the game. He rolls again and moves his pawns according to the numbers showing on the three dice. After the first roll, the players throw the dice and alternate turns. The roll of the dice indicates how many points, or pips, the player has to move his pawns. The pawns are always moved counter clockwise to pass from the own home-board to that of the players on the right side and finally reach the opposite one.
The picture shows the direction of movement of Black and Red

A pawn may be moved only to an open field, one that is not occupied by more than one opposing pawns.

A player can move his pawn in the opponent's quadrants only if he has put all his pawns in his own quadrant. Then he can move toward the right side.

The numbers on the three dice constitute separate move or they can be added. For example, if a player rolls 2 , 3 and 5 he may move one pawn five spaces , another pawn three spaces and another one four spaces. Or he can decide to move 2+3=5 spaces and the second pawn the remaining 5. Otherwise he can choose to add 2+3+5 =10 and move only one pawn 10 spaces.

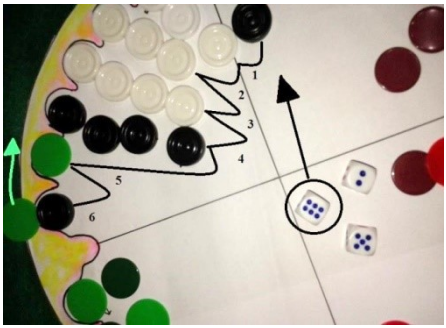
A player who rolls doubles plays the numbers shown on the dice twice. Furthermore a roll of 6 and 6 means that the player can add a pawn in one of his own field (if needed).

With the triple: in this case the player removes a pawn from each opponent from the fields, and if they haven't any, he puts his own pawns in those fields.

With the order: in addition to the normal rule, the player can move, if it is convenient, only one pawn according to the higher or the lower number on the dice.

When neither number can be used, the player loses his turn. In the case of doubles or triples, when all four/six numbers cannot be played, the player must play as many numbers as he can.

2nd Phase: Hitting and Entering the Pawns



A point occupied by a single pawn of either colour is called a hurt. If an opposing pawn lands on a hurt, the hurt is hit and given back to the owner who has to start the move from the point 1 – the beginning.

A pawn can land only in an empty point or on a hurt; otherwise it can only pass over.

Any time a player has one or more hurt pawns back, his first obligation is to enter those pawn(s) into his own home board before moving the other pawns that are already on the board. A pawn is entered by moving it to an open point corresponding to one of the numbers on the rolled dice.

For example, if a player rolls 4, 6 and 2, he may enter a pawn onto the opponent's four points, six points, two points, ten points, eight points or twelve points, so long, as the prospective point is not occupied by two or more of the opponent's pawns.

If neither of the points is open, the player loses his turn. The roll has to be used by the next player and if it is not possible by next one. The player, who has gained this turn, may also roll the dice in addition. If any player cannot use the move, the turn is lost.

If a player is able to enter some but not all of his pawns, he must enter as many as he can and then forfeit the remainder of his turn. After the last pawns of a player have been entered, any unused numbers on the dice must be played, by moving either, the pawn that had been entered or a different pawn.

3rd Phase: Bearing off the Pawns

Once a player has moved all of his twelve pawns out of his home board, he may start the bearing off. A player bears off a pawn by rolling a number that corresponds to the point where the pawn can get through the last pip on the right side of the opponent quadrant.

In this case the roll is 6-5-2

The player decides to bear-off two pawns: the first by using 6, the second adding 2 and 5 = 7.

If a pawn is hit during the bear off process, the player must bring that pawn back to his home board before continuing to bear off. The first player to bear off all twelve pawns wins the game.

Remarks

The dice must be rolled together and land flat on the surface of the board. The player must reroll all dice, if a dice lands outside or lands on a pawn or does not land flat. If a player rolls before his opponent has completed his turn by picking up the dice the player's roll is voided.

Game Instruction and a Game Example

Four Seasons: https://youtu.be/7nbBY8XrP0I

YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

WHICH MATHEMATICAL CONTENT CAN BE LEARNED?

NUMERACY

- Count reliably up to 20 items
- Interpret +, - and = in practical situations for solving problems
- Multiply using single-digit whole numbers
- Use a calculator to check calculations using whole numbers
- Order and compare numbers up to 40
- Understand the concepts of double and triple
- Add single-digit numbers with totals to 20
- Determine a next term in linear patterns (e.g., 2, 4, 6…)
- Relate familiar events to the year, month and week
- Understand probability
- Identify the range of possible outcome when using more dices
- Determine the approach, materials and strategies to be used
- Develop the mental calculation abilities

GEOMETRY

- Recognize and name two dimensional shapes (circle, circular sector)
- Understand symmetry in shapes
- Understand and compare angles
- Understand concepts of diagonal, horizontal and longitudinal lines

HISTORY

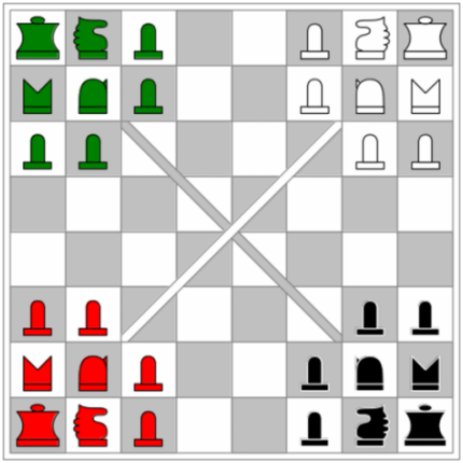
One of the first European books on games was Libro de acedrex, dados e tablas, commissioned by the Spanish king Alphonse X of Castille, Galicia and León. It was completed in his scriptorium in Tole-do in 1283. The book contains several interesting historical games, including Four Seasons. It was written to collect the main pastimes and entertainment of those ages.

As its name implies, the game is a competition between four players, each representing one of the seasons. Moreover, each player's colour represents one of the four elements and a mood. Green represents spring, air, and blood; Red represents summer, fire, and choler; Black represents autumn, earth, and melancholy; White represents winter, water, and phlegma.

The structure of the "dashboard" is essentially that of "backgammon" but the boxes are arranged in a circle in order to facilitate access to the four involved players. A special version of the most widely used "tabula" in the Middle East dates back to 4,500 years ago.

VARIANT

FOUR SEASONS CHESS



Four Seasons Chess is played by four people on a modified chess board. The main diagonals of the board are marked in the middle, to help make the moves of the pawns clear.

Each player starts with a king, a rook, a knight, a bishop and four pawns. As pawns can become queens in the game, some way of marking the pawns should be kept in hand (e.g. coins to place underneath).

1. At the beginning of the game the pieces are set out with green (spring) in one corner, red (summer) to his right, then black (autumn) followed by white (winter). This is shown in the diagram.
2. Player decides at random who will take each side.
3. Spring begins the game, with play passing anti-clockwise around the board in the same order as the seasons.
4. In his turn a player moves a single piece according to the following rules:
    - A pawn may move one step forwards in the direction that it has faced since the start of the game (roughly along the board edge).
    - The rook moves as many squares as is convenient in one of the four horizontal or vertical directions, though without jumping over any piece in the way.
    - The knight moves one square horizontally or vertically, and one further diagonal step away from its current square, jumping over any piece in its way.

- The bishop moves exactly two squares diagonally, jumping over any piece in its way.
- The king moves one step in any direction, horizontally, vertically or diagonally.
- A queen (see rule 6) moves one square in any diagonal direction.

5. Only one piece can occupy a square at a time.
6. A pawn reaching the far row of the board is promoted to a queen.
7. A pawn may capture an opponent's piece by moving one square diagonally forwards to land on its victim. The captured piece is removed from the board.
8. Other pieces capture by moving in their normal manner to land on the victim.
9. A king cannot be captured, but may be threatened with capture. A king so threatened must on his next turn deal with the threat in one of the following ways:
   - by moving the king out of danger
   - by capturing the piece that poses the threat
   - by interposing another piece between the king and the piece that threatens him
10. If a player's king is threatened and he cannot escape as specified in rule 9, then the player has been checkmated and drops out of the game.
11. The conquered player's king is removed from the board and his pieces fall under the control of his conqueror.
12. The last player remaining in the game is declared the winner.
13. It is usual for each player to attack the player to his right and defend against the player to his left, but the rules of victory do not enforce this. This order may, however, be used to determine who is the conqueror in rule 11 if two players simultaneously checkmate another.
14. Though usually played without dice, the game can be played with an ordinary six-sided die, rolled before each move.
15. A throw of 6 dictates the king be moved, 5 a queen, 4 the rook, 3 the knight, 2 the bishop and 1 a pawn.
16. If the player has none of the specified pieces on the board, the turn is lost.

References and Links

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http://ludus123.blogspot.it/2015/07/four-seasons-chess.html

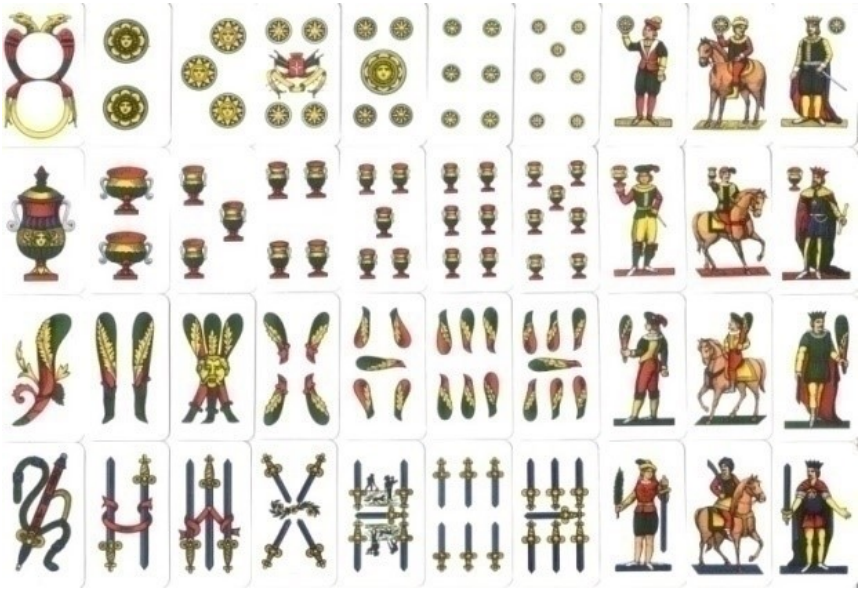
7.3 Steal the Pile

You can play the game on the table, on the floor, wherever there is a board. And you need game cards.

Overview

- **Genre:** card game
- **Players:** 2, 3, 4 players
- **Age range:** older than 6
- **Setup Time:** 1 minute
- **Playing Time:** 3 minutes to 6 or 7 minutes
- **Random Chance:** no dice needed

Game Rules



This popular game is for 2, 3 or four players. It is played by cards. A deck of cards from Naples is used in Italy. They have different suit marks: swords, coins, batons and cups. Each deck consists of 40 cards made ordered from 1 to 10 of each suit. The last 3 numbers 8, 9 and 10 are represented respectively by an infantry-man, a horse and a king.

The intention is to have more cards then the opponents at the end of the game.

The Game Proceeds in Three Phases:

- 1st phase distributing the cards
- 2nd phase shooting cards
- 3rd phase stealing the pile.

1st Phase: Distributing Cards

The game begins by shuffling the cards. Then each player picks a card out of the pack. The player who has the highest card will be the dealer. He will shuffle the cards, the player on the left will cut the cards and the dealer will distribute the cards, giving three cards to each player counter clockwise, starting from the player on the right. Then he will put four cards face up in the centre of the table.

Picture 1

Picture 2

2nd Phase: Picking up Cards

Players alternate moves. Now the game can start. The first player picks up a card. As the aim of the game is to conquer the more as many cards as possible, he tries to take them from the table. But he may take only the same card he already has, so for example if on the table there is a three, and if the player has a three in his cards, then he can take it. Or he can take more cards, where the sum is the same number he has himself. For example if he has a horse, which is nine, and on the table there are a two and a seven, he can take both with his nine (Picture 1).

The taken card must stay face up on the table, on the top of the pile of cards in front of him and in the second case the card which is shown on the top of the pile must be the higher one (Picture 2).

If the player hasn't got the same cards that are on the table, he has to add one of his cards on the table. After three shots, each player will have no cards anymore. So the dealer will distribute three more cards to each one. This will continue until the cards of the deck are used up.

3RD PHASE: STEALING THE OTHER'S PILE

Each player can "steal" the pile of any opponents if he has the same card which is on the top of it. In this case he will shoot the card on the table and he will take the opponent's pile where the same number is, adding it to his own pile.

At the end of the game the players will count their cards and the winner will be the one who has got more of them.

STRATEGY

There is not a big strategy, the game is very simple. But in case a player has in hand, for example, a seven and two fours, and on the table there is a seven, it may be convenient not to play the seven but the four. If the opponent has a four too and takes it, the first can steal the pile with his second four.

MORE INFORMATION AND A GAME EXAMPLE

Links to google play where it is possible to download the game for mobile phone

https://play.google.com/store/apps/details?id=it.dt.rubamazzetto.ui

https://play.google.com/store/apps/details?id=nndroid.rubamazzo&hl=it

A game example video of "Steal the Pile" https://youtu.be/JoX2zz5G2cE

YouTube: https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

WHAT MATHEMATICAL CONTENT CAN BE LEARNED?

NUMERACY

- Count reliably up to 10
- Order and compare numbers up to 40

- Add single-digit numbers with totals to 10
- Develop easy strategy skills
- Sort and classify objects using a single criterion
- Understand probability
- Identify the range of possible outcome when using a card
- Determine the approach, materials and strategies to be used
- Develop the mental calculation abilities

History

It is told that cards have originated in China and then spread to India and Persia. From Persia they have reached Egypt during the era of Mameluke control, and from there into Europe through both the Italian and Iberian peninsulas in the second half of the 14th century. The Mameluke suits were goblets, gold coins, swords, and polo-sticks. Polo being then unknown in Europe, these were transformed into batons or staves, which, together with swords, cups and coins, are still the traditional suit marks of Italian and Spanish cards.

In the fifteenth-century German card-makers experimented with suits vaguely based on Italian ones, eventually settling for acorns, leaves, hearts and bells (hawk-bells), which still remain in use. Around 1480 the French started producing playing-cards by means of stencils, and simplified the German shapes into trefle (clover), pique (pike-heads), coeur (hearts), and carreau (paving tiles). English card-makers used these shapes but varied the names. Spade (pique) may reflect the earlier use of Spanish suitmarks, from espadas meaning swords, and clubs are what the Spanish suit of staves actually look like. Diamond is not only the shape of the paving tile, but may perpetuate connotations of wealth from the older suit of coins.

Playing cards have always served two distinct purposes: gambling, and the playing of games of skill. Their introduction provided a new alternative to chess and draughts, dice and knuckle bones. Sometimes they also provided a new way of telling fortunes.

The first European references to playing cards date from the 1370s and come from Catalonia (Spain), Florence, France, Sienna, Viterbo (Italy), southern Germany, Switzerland and Brabant. Most of these refer to 'a recent introduction'.

No cards from this early period have survived, but the sources indicate that cards were being painted 'in gold and various colours' or 'painted and gilded' which suggests luxury packs. The earliest surviving cards are from the fifteenth century, and most of these were made on pasteboard manufactured from 3, 4 or up to 6 sheets of paper glued together. Cards were often much larger sizes than today, and the images were either hand drawn or printed from woodblocks or printed from copper engravings. The colouring was often done using stencils.

In the ancient time the card game was called naibi in Italy and naipes in Spain: the word comes from the Arabic meaning na'ib representative of the king, one of the figures who made up the deck and some say that would correspond to what is now the knave. Italy is certainly the country where the cards have had the greatest development both as production as well as artistic quality, that's why Naibbe could be a corruption of the name of Naples.

In Italy Steal the pile is one of the first games that a child learns, on the one hand due to the simplicity of the rules and on the other because people are considered to like stealing from birth.

Variant

It is possible to play catching cards from the table simply by having a card of the same suit. To steal the pile, the player needs a card of the same suit.

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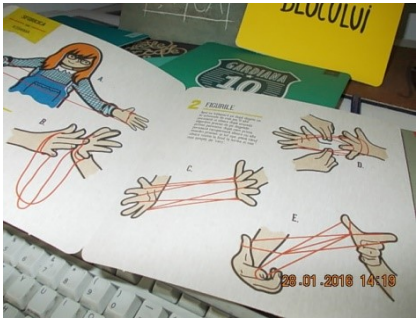
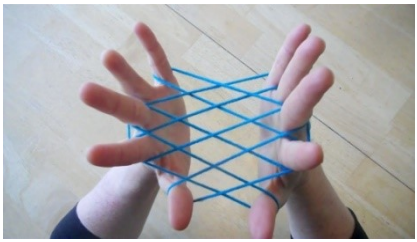
http://www.theguardian.com/notesandqueries/query/0,5753,-2647,00.html

http://www.scudit.net/mdcartestoria.htm

8.1 The Crow's Feet (String Figure)





Overview

The name of this game is "The Crow's Feet" or the "Little String". Other names are: Cat's Cradle, Counting, the guardian 10, the little line, the Little Square, the Elastic and String Figure.

This game can be played outdoors, indoors, at school, during the break; it can also be played by the adults who want to teach their children a new game. This game for two players is extremely easy, but the players need abilities, intelligence, patience, concentration and calmness. You play it with strings in your hands. The game is very famous to train these abilities.

Stages of the Game

Stage 1: They take a 70 cm long piece of string and tie its ends.
Stage 2: The 1st player coils it round 2 fingers.
Stage 3: The second player takes over the string with his fingers using certain precise movements.
Stage 4: The 1st player recovers the string with his own fingers.
Stage n: The stages are repeated until the end when the string must unwind.
End of the game: It is played in the initial circle form.

Vocabulary

„Outch! Don't pull, I've caught my fingers!''

Examples, References and Links

https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

https://www.youtube.com/watch?v=zlHfMkxVInU

https://www.youtube.com/watch?v=o-ekqfSz428

http://www.infatablocului.ro/ http://www.wikihow.com/Do-String-Figures
https://youtu.be/AIIAZz37dYQ https://youtu.be/FyTi7Pf7LXk

https://youtu.be/KNDErjr2p6c https://youtu.be/Vb6DWj4OLd0

See "Mendebilul". Mircea Cărtărescu, Nostalgia, Humanitas, anniversary edition, 2013

What Mathematical Content can be Learned?

Mathematical Literacy

- Count reliably up to 10 items
- Recognize and name two dimensional shapes (for example circle, rectangle, parallelogram, diagonal lines, triangles, angles)

Remark of Recent History of the Game "Made in Romania"

by Georgeta Adam

The other day I read a message on Facebook: "When I was a kid the social network was called «Outdoors»". We do not know for sure the age of the person who posted it, but we might assume that now he must be between 30 and 40 years old or more. As the message is in Romanian, using the mathematical logics, we can say that he hails from Romania. I have lately been acquainted to two young people - Vlad and Mihai - belonging to "the key around the neck generation", which means from Romania, Eastern Europe, where the parents living in towns were necessarily involved in a "work activity", but even so, they lacked the means to hire nannies.

What did these children play and where? Outside their block of flats! Resourcefulness, creativity, joy to communicate using whatever they had at hand, such as an elastic, a piece of string, a ball, a rope and all this became were turned into intense playing! 55 such games have been revived of late by way of championships-meetings between parents and children and then included in the games box entitled Outside the Block of Flats. The book with picture postcards briefly explains the games of the "communist age", aiming at restoring for today's children the "box childhood" of their parents. The book emphasizes the link between generations, the intention of getting the contemporary Facebook generation acquainted with these simple, but lively, spontaneous, social games, open to all those who gathered in open air to live a colourful childhood and escape the greyish social background.

The author has selected just a few games "Made in Romania", which rely on mathematics, using elements such as the circle, the rectangle, numbers, as she has them in her mind due to hers and hers son's childhood, who was eager to play all sorts of games.

8.2 Skipping Rope

Overview

- **Genre:** outdoor games, urban, rural environment
- **Players:** 4-5 at the 10-to-1 variant and at the skipping rope in 3 variants: 10-15 players
- **Age range:** 5 years and up
- **Setup time:** 1 minute
- **Playing time:** about 30 minutes
- **Random chance:** no
- **Skills required:** orientation abilities, counting while jumping, strategical change

Game Description

This game requires a skipping rope (a thicker sort of string made of hemp; they are to be found in the toy shops). The game implies imaginative memory, high speed, dexterity and ability in creating authentic shows. The girls proved to be more gifted than the boys and they love practicing it from childhood to youth. Even adult persons like to practise it.

The player must correctly perform all the 10 different series of jumps in a decreasing order from 10 to 1. When one fails, another player's turn comes. When his/her turn comes again one resumes the jumps from where one has stopped them.

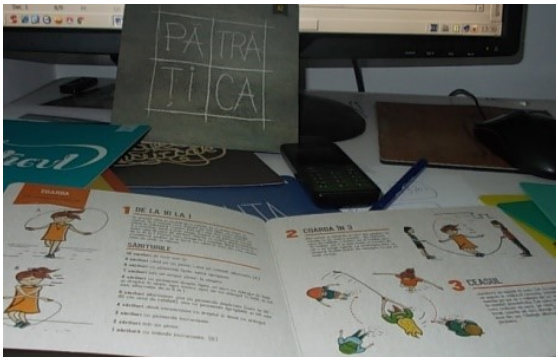
There are various ways of deciding the winner.

The winner can be considered the one who has achieved the greatest number of jumps without failing.

ADVICE

Do not jump too high, but just as high as to allow the rope to pass under your feet. Use your wrists to move the rope, not the shoulders while keeping the elbows close to the body.

Alternate the spinning directions lest you should feel giddy when practising the „Clock" variant.



Stage 1: It starts from 10 skips, performed according to each player's choice
Stage 2: 9 skips, jumping alternatively with the right leg, with the left leg
Stage 3: 8 skips, the players jump on two legs, kept close
Stage 4: 7 skips on one leg, at the player's choice
Stage 5: 6 skips keeping the legs straight and close, first pushing the left forwards and the right behind and then the other way round
Stage 6: 5 skips alternating open and close legs
Stage 7: 4 skips, 2 times on the right leg and two times on the left leg
Stage 8: 3 skips with crossed legs
Stage 9: 2 skips on one leg, at one of them crossing the arms
Stage 10: 1 skips with crossed arms

Variants

There are variants in which at the stage 5 (6 jumps), the player says: „green leaf wait for 6, /I've done 6!"(In Romanian this distich rhymes and it has a mnemonic function).

There is also a variant in which at stage 9 (2 skips), the player says: „green leaf of clover/I've done 2". This is also a rhymed distich.

Other specific vocabulary elements at the Clock are: You failed / Fail / Miss", "I was at 5! You were not, you were at 7! Give me the rope back, I go home!"

Skipping rope in 3: There is a variant in which two players stretch the rope at about 150 cm distance and a third player jumps. More players may jump provided that the rope is longer. The Americans call it „Double Dutch".

The Clock variant: The players stand in circle, the one with the rope stands in the middle of the circle. He will turn round the rope at the earth level and the players of the circle will skip in turn, without touching the rope. The one who does is expelled from the game. The winner is the last standing player.

What Mathematical Content can be Learned?

Numeracy

- Count, add and subtract single-digit numbers up to 10

Geometry

- Recognize and name two dimensional shapes (circle, semicircle)
- Make statistics and understand simple information from lists and tables

References and Links

http://www.infatablocului.ro/

https://en.wikipedia.org/wiki/Skipping_rope

Youtube: https://www.youtube.com/watch?v=thLXlw0y1P8

8.3 Hora (Romanian Round Dance)

Overview

Hora from Fundatura, Vrancea, Romania

- **GENRE:** traditional folk dance
- **PLAYERS:** a small number which can increase to several tens or hundreds of people
- **AGE RANGE:** from young to elderly people
- **SETUP TIME:** from a couple of minutes to the end of the song time
- **PLACE OF PERFORMANCE:** on the field, in the village, on a special "hora" place

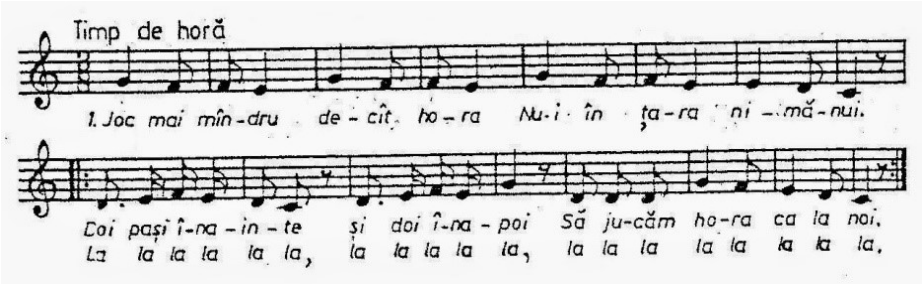
Game Rules (Dance Description)

General

Hora (the round dance) is a Romanian traditional dance which assembles everybody in a big circle. The dance is accompanied by musical instruments, such as the cembalo, accordion, viola, violin, saxophone, clarinet, trumpet or panpipe. The ethnographic works talk in eulogistic terms about the round dances from Banat, Transylvania, Oltenia and Moldavia, which have distinct features, but are similar in what is essential and specific to the Romanian soul. Hora is the link between the cult of the Mother Goddess, specific to the archaic matriarchal societies and the cult of the Sun, represented by circular shapes, the round dance being itself a living circle. The origin of the round dance is not known, the dance coming down from generation to generation for thousands of years. The circular shape of this dance points to the fact that the circle, as a cosmic formula, represents a closed universe, that circular shapes are to be found in art too and the vibration mass of the sound has the same shape.

STRATEGY

It is a peasant dance that assembles the whole audience in a big circle. The dancers hold hands and step either diagonally or forward or backward, at the same time moving the circle, generally clockwise. In its classical form, each dancer takes three steps forward, one step backward, left and right.



The big round dance, score and lyrics

Stage 1: The stage 1 is the coming of the musicians, who sit in the middle of the festivity place or on one side of the stage.

Stage 2: The stage 2 is the coming of the girls, lads and of the onlookers; there is also the free hora entrance, as a line of girls, lads, pairs of dancers or as a spiral of dancers.

Stage 3: In stage 3 the music starts, girls are invited to dance, they hold hands.

Stage 4: The round dance begins. Dancers tread some measures to the centre of the circle, and then backwards, moving the hora clockwise, which is the direction of the sun movement.

Stage 5: The round dance develops, getting bigger and bigger with new pairs, girls, lads, women and men joining the circle.

Stage 6: Some pairs detach themselves from the circle and get on dancing in its inner space; polka - a detached boy invites a girl to dance;

Stage 7: Variants: hora with chanted verse, with whistles, hora with a soloist singer who interprets a topical song;

Final stage: The moment music stops; after a few moments or immediately the players start another tune and the dance goes on.

HISTORY

The origins of the round dance can be traced back to the Thracian civilization, to the proto-Dacians. The name of the dance comes from the Greek word “choros”, which had been borrowed from the Thracians. In 1942 on the Cetatuia hill in Boldestii de Jos village, Neamt, Romania, a ceramic representation of the round dance performed by six women was discovered. The object belongs to the Cucuteni culture (3,700-2,500 B.C.), which proves that the’ hora” appeared in Dacia before 5,000 B.C. It is a masterpiece of the Cucuteni culture, named “Hora de la Frumusica” (The round dance from Frumusica), consisting of an anthropomorphic group of statues representing women joined in the movement of an archaic round dance. The round dance was mentioned for the first time in modern times in the 17th century by Martin Opitz de Roberfeld (1597-1639), who is considered to be “the father and restorer of German poetry”. In 1622 he was a teacher at the lower secondary school from Alba-Iulia, acquainted with the Romanian realities, ceramics, costumes and language of the oldest inhabitants of Transylvania. He warmly presented the Romanians and the Romanian ”hora” in his poem “Zladna oder von der Ruhe des Gemütes” (Zladna or the Inner Peace). The first native scholar who wrote about the round dance was Dimitrie Cantemir (1673-1723), a famous humanist, in his book Descriptio Moldavie (1717), in which he mentions the musicians as well.

The round dance from Frumusica, Romania, the

Hora from Topalu, Dobzogea, Romania (1970)

The “peoples’ spring” in the 19th century boosts the study of the national cultures. Two years after the revolutions of 1848 , Anton Pann composed 11 tunes on a hora rhythm, in “The Love Hospital or The Singer of Desire”. In

1918 the Hungarian musicologist Bela Bartok publishes “Romanian Folk Dances” and “Romanian Folk Music from Maramures”.

Round dance from Transylvania, Prundul Bârgăului

The Round Dance and its Cultural Impact

The round dance left lasting traces in the Romanian onomastics, literature and art. An example is the name Horea, the name of the Transylvanian leader of the anti-feudal uprising of 1784, who was killed by being stretched on a wheel in Alba-Iulia in 1785. The name originates in the same “hora”, an event during which Nicolae Urs alias Horea used to play the “flute of Albac’s inhabitants called moti”. In the songs of that time Horea’s Uprising is compared to a fire around which the hora of the three captains (Horea, Closca and Crisan) was revolving. “Join our hora and dance in the light of the fire” are the lyrics of the song. The “bard “ of Mircesti, Vasile Alecsandri (1818-1890), wrote his famous poem “The Unity Round Dance”, which Mihail Eminescu appreciated as being “the most beautiful round dance of the Romanian nation”.

Its lyrics were put on music by the composer Alexandru Flechtenmacher and they were sung and danced in the great squares of Bucharest and Iasi when the Prince Alexandru Ioan Cuza ascended to the throne (1859-1866). At this moment the round dance became the symbol of the Romanian brotherhood. On the 24th of January 1859 the song and the dance merged in harmony with that feeling. “Let’s hold hands,/ Us, with a Romanian heart,/To dance the brotherhood round dance,/ On the Romanian land”. In Romulus Vulcanescu’s opinion,” hora” represents a dominant feature of the Romanian people. Studying the versatility of

Round dance from Dobrogea in Chisinau

Theodor Aman - The Unity Hora from Craiova

the round dance in the Romanian choreography, the folklore expert C. T. Niculescu Varone identified 5,332 dances called hora. George Cosbuc in his poem "Zamfira's Wedding" describes the dance in memorable verses that flow in the hora rhythm itself: "Three slow steps to the left/ And other three steps to their right, /They hold hands which they detach/ They join in circle and stretch it, /And tread the earth in easy tact." The novelist Liviu Rebreanu describes the horas of the Transylvanian village in his novels "Ion' and 'Ciuleandra".

Across the Carpathians, in the Oltenia province capital Craiova, the painter Theodor Aman painted the wonderful round dances in his famous canvas "The Aninoasa Hora".

The Romanian hora is perhaps the oldest dance historically known and attested since ancient times, which deserves a place in the world spiritual heritage. In January 2016 the Romanian dance "Feciorescul de Ticuş" (Lads Dance) became part of the UNESCO spiritual heritage of the world.

VARIANTS

Hora, alongside with Sarba, Batuta, Braul, Invartita, Trei lemne etc., belongs to the category of collective dances which reflect the emotional energy of the community. It consists of two archaic essential types: a) the closed hora, of the circle type and b) the open hora, of the spiral type. The former is danced at all the ritual, ceremonial and festive occasions, while the latter is danced only ritually and ceremonially. Romanian folklore experts emphasize that the round dance was initially a symbol and a religious dance expressing a telluric mythological experience. For the contemporary dancers, this significance lost its meaning; it is

only the choreographic technique that survived. At present the round dance has a lot of variants. Here are a few:

- batuta (in 2/4 measure, danced especially by men with lively movements);
- free horas – danced by men and women, the dancers forming a circle or a half circle, holding hands;
- the ostropat hora – bridal rituals, in 7/16 or 3/8 measure, the participants, with a dowry object in hands, dance vividly and individually in front of the wedding guests;
- the bride's hora or the godparents' hora - a peasant dance from Banat, Oltenia, Muntenia, Dobrogea and Moldavia. It is danced only at the weddings, at the bride's house, before her leaving for the groom's house.

Hora from Dobrogea, Romania

The Village Museum Hora

Hora from Vintileasa, Vrancea, Romania

The godparents start the dance with the newlyweds and shortly after the guests joins them. Hora is danced taking one step to the right and then one step to the left. Three steps to the right follow and then five steps to the left and two earth treads. In the country this dance lasts until a "colacer' or "vornicel" helps all the wedding guests, besides girls, to a drink

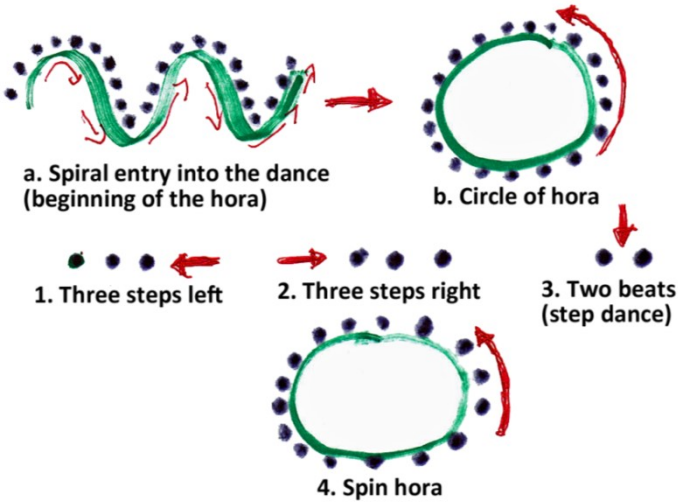
from a “plosca” full with wine, the vessel being pyrographed and decorated with flowers and Romanian national colours ribbons.

“hora-mare” - in 6/8 or 3/8 measure with moderate slow movements, in circle, the dancers hold their arms bent from the elbows and rose to the shoulders level.

In the Middle Ages the so-called “Peasant dance” was very much disseminated, taken over by the nobles as well, being turned into a folk dance.

The round dance can be named according to the place of origin - “The Hora from Orhei” - or it can bear the name of the person or event in whose honour it is performed, “The Bride’s Hora”, “Ilenuta’s Hora”, “The Wedding Hora”. On some occasions the word hora is omitted and the dance is simply called” Nuneasca”, “Floricica”, etc.

Phases of Hora



What Mathematical Elements Are in This Dance?

Numeracy

- Understand a coordinate system
- count reliably up to 10
- Know the follower and the predecessor of a one-digit number

Geometry

- Recognize and name two dimensional shapes

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Romulus Vulcanescu, Mitologie română (Romanian Mythology), Academy Publishing House, Bucharest, 1985

Lucian Predescu, Enciclopedia Română Cugetarea (Romanian Encyclopedia Cugetarea), Saeculum Publishing House I.O, Vestala Publishing House, Bucharest, 1999

LINKS

http://enciclopediaromaniei.ro/wiki/Theodor_Aman

http://clasate.cimec.ro/Poza.asp?tit=Pictura--Aman-Theodor--Hora-unirii-la-Craiova&k=C5AB940805C2431782653962E2835646

https://commons.wikimedia.org/wiki/File:Theodor_Aman_-_Hora_Unirii_la_Craiova.jpg

http://www.bistritaculturala.ro/stire.php?id=39

https://ro.wikipedia.org/wiki/List%C4%83_de_dansuri_populare_rom%C3%A2ne%C8%99ti

http://www.juniisibiului.ro/

http://www.infotravelromania.ro/fotografii_traditiiromanesti.html

YOUTUBE

https://www.youtube.com/watch?v=oVrVjfYJKSc

Floarea Calotă: Haida, haida, frăţioare! (Come on, come on, brother!) Despite the interpreter and the song (as well as the clothes of most dancers) being from Teleorman-Wallachia (near Bucharest), there are costumes from several historical provinces in this hora.

9.1 Fifteen Game

The 15-puzzzle is a game of sliding pieces that have a specific initial order within a square box.

Overview

- **Genre:** sliding puzzle
- **Players:** 1
- **Age range:** from 8-9 years old
- **Setup Time:** less than 1 minute
- **Playing Time:** depending on the ability of the player
- **Random Chance:** none

9	8	3	
1	15	5	11
7	12	9	4
10	6	2	13

Game rules

General

The “15 GAME” is a sliding puzzle that consists of a frame of numbered square tiles in random order with one tile missing. The tiles are numbered from 1 to 15. The object of the puzzle is to place the pieces in order by making sliding moves that use the empty space.

12	1	2	15
11	6	5	8
7	10	9	4
	13	14	3

The goal is to reposition the squares from a given arbitrary starting arrangement by sliding them one at a time into the configuration shown above (for some initial arrangements, this rearrangement is possible, but for others, it is not).

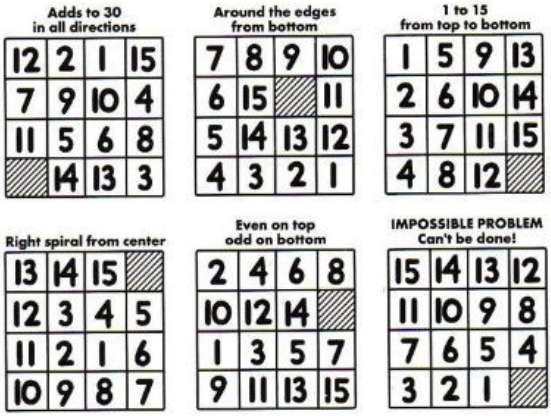
The initial position

The fifteen pieces should be placed at random in the 4x4 square box, leaving an empty space like in the figures.

Then you should sort the pieces in ascending order without taking the pieces out of the box (so the only thing that is allowed is to slide the pieces).

Example: https://www.youtube.com/watch?v=qFzx1nzDyCY

OTHER PATTERNS



There are more than a trillion possible combinations of the numbers 1-15. These are some samples of patterns one can try to solve.

But not every randomly placed pattern of pieces can be sorted by just shuffling. Johnson & Story (1879) used a parity argument to show that half of the starting positions for the 15-puzzle were impossible to resolve, no matter how many moves were made. This is done by considering a function of the tile configuration that is invariant under any valid move, and then using this to partition the space of all possible labelled states into two equivalence classes of reachable and un-reachable states.

Sam Lloyd's unsolvable 15-puzzle, with tiles 14 and 15 exchanged. This puzzle is not solvable because moving it to the solved state would require a change of the invariant.

What Mathematical Content can be Learned?

Numeracy

- Count, read, and write whole numbers up to 20
- Order and compare numbers up to 20
- Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 2, 4, 6...; 15, 14, 13...)

Mathematical Reasoning

- Determine the approach, materials, and strategies to be used
- Use tools, such as manipulative or patterns, to solve problems

Geometry

- Recognize and name two dimensional shapes (square, rectangle)
- Describe length and width of shapes
- Measure areas by counting squares

History

The game has its origins in the United States in the nineteenth century. The game has long been attributed to Samuel Loyd in the United States in the late 1870s, however, sources say that the true author was Noyes Palmer Chapman.

It is not clear when the first slide puzzle was invented (or made). But it is a known fact that in 1878 Sam Loyd, America's puzzle-expert, "drove the whole world crazy" with his newly "discovered" 14-15 puzzle (though some accounts state that, other authorities dispute this fact). Sam Loyd claimed from 1891 until his

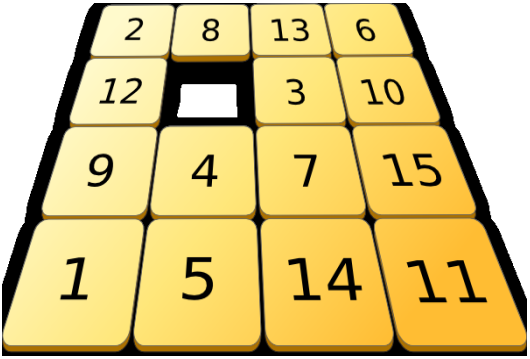
death in 1911 that he invented the puzzle. Actually, he had nothing to do with the invention or popularity of the game... This was a variation on the "15 Puzzle" which was made and sold by the Embossing Company from New York about 10 years earlier.

It was not surprising that Sam Loyd drove the whole world crazy by his variation of the puzzle of 15. The problem that he formulated was impossible to solve. When you bought Loyd's 14-15 puzzle the empty square was positioned bottom right. The pieces were numbered in order from left to right and from top to bottom; only the pieces numbered 14 and 15 were reversed. You should re-order the pieces so all the pieces are in the correct position and the empty place should be positioned bottom right again. A slide puzzle with square pieces can only be solved when the number of exchanges necessary to solve the puzzle is even.

Between 1880 and 1882, the 15 puzzle became a real social plague that swept the US and Europe. However in 1882, it was discovered that of all the problems posed in the game, only half had solution. Advertisements in newspapers rewards to anyone who solved at least one of the unsolvable problems and various New York newspapers offered $ 1,000 prize for anyone who could provide a solution for achieving a particular combination specified by Loyd, namely reversing the 14 and 15. Nobody won. The game ended up disappointing the population and thus ended 15-game fever.

Synonyms

The 15-puzzle is also called Gem Puzzle, Boss Puzzle, Game of Fifteen, Sliding Puzzle, Mystic Square, Le Taquin, etc.



Variants

The puzzle also exists in other sizes, particularly the smaller 8-puzzle. If the size is 3×3 tiles, the puzzle is called the 8-puzzle or 9-puzzle, and if 4×4 tiles, the puzzle is called the 15-puzzle or 16-puzzle.

Many fun and promotional versions of the 15-puzzle have been made with graphics of some kind rather than numbers. Sometimes the graphics are such that every square is unique, and so it is really exactly the same puzzle as the standard 15 Game.

A nice variation of the basic slide puzzle is a slide puzzle with 15 pieces numbered form 0 (empty space) to 15 with the instruction to slide the pieces so that the sum of all rows and the sum of all columns and the sum of the diagonals equals 30. Such an ordering is called a magic square.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/15_puzzle

https://es.wikipedia.org/wiki/Juego_del_15

Fifteen Puzzle: http://www.cs.brandeis.edu/~storer/JimPuzzles/ZPAGES/zzzFifteen.html

The history of the 15 puzzle: http://hc11.home.xs4all.nl/15puzzle/15puzzen.htm

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http://15-slide-puzzle.software.informer.com/

https://itunes.apple.com/us/app/15-puzzle-sheep-free-classic/id1016289760?mt=8

http://download.cnet.com/15-Puzzle/3000-2111_4-10017018.html

https://play.google.com/store/apps/details?id=com.nonnogame.fifteenpuzzlemania

9.2 Seven and a Half

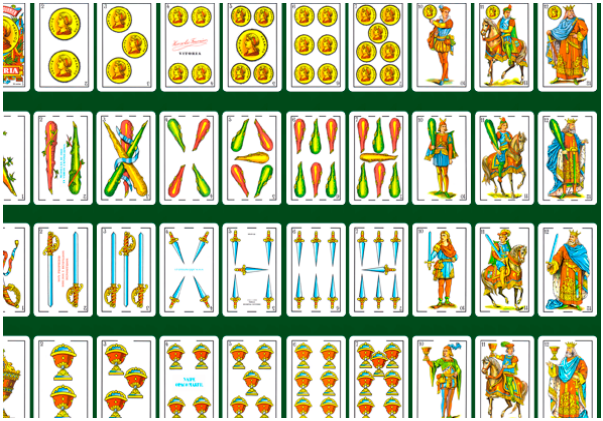
This is a Spanish card game, played with the standard Spanish deck of forty cards.

Overview

- **Genre:** card game
- **Players:** any number of players may take part in the game, but it is not recommended to be more than 8
- **Age range:** older than 5
- **Setup Time:** about 30 seconds
- **Playing Time:** 1-2 minutes
- **Random Chance:** rather high

Game rules

General



The Spanish pack has 40 cards distributed in four suits: golden coins, glasses, swords and sticks; the picture cards are jack (J or 10), horse (Q or11) and king (K or 12).

Any number of players may take part in the game, but it is not recommended to be more than 8. Traditionally players bet small amounts of chickpeas or pebbles on every hand and there is no any specific goal to win a whole game. Each player competes against the bank; the goal is not to exceed the sum of seven and a half or a closer number.

Picture cards have a value of half a point each and the values of the remaining cards are their indexes. In some varieties, the ace or 1 can value their index, one point, or half a point, depending on the needs of the player.

How the game goes on

The bank is assigned randomly to any player at the start of the game. All players receive one card at the start of the deal.

All players, starting from the right of the bank, must perform two actions:

- First they must announce the amount of points to bet; usually this amount is limited to a certain range.
- Ask for more cards, one by one, or just stand at any moment.

Only one card may be kept face down; when asking for a new card, there are two options:

1. Ask for a new card face up.
2. Ask for a new card face down; in this case the card that previously was hidden must be put face up.

When the sum is already greater than seven and a half (the player has passed), all the cards must be shown and the bet is immediately lost.

The player who holds the bank must play after the others; if all of them have passed, the bank does not need to play. The banker does not have to bet, and must play as the other players, trying to get seven and a half or a near result.

The bank plays against all the other players; so, if the bank has passed, all the other players who have remained win their bets.

If the bank stands, the cards are compared. The goal is to reach the nearest amount of points to seven and a half. If both have the same amount, the bank wins; in particular, if the bank gets seven and a half, there is no need to view the cards of the remaining players.

The bank pays the bet to the players who have won, and the other way round.

The player who gets seven and a half, if the bank does not get it, is paid double the amount, and becomes the new bank in the next deal.

Before starting a new deal the bank may be sold to any player who makes an offer to buy it.

What Mathematical Content can be Learned?

Whole numbers

- Count reliably up to 20 items.
- Read numbers up to 20.
- Add single-digit numbers with totals to 20.
- Use a calculator to check calculation using whole numbers.

Fractions, decimals and percentages

- Read and add halves of quantities.

Probability

- Understand probability in order to decide whether to ask for another card or not.
- Identify the range of possible outcome when using a card by remembering the cards that have been played.

Synonyms

Siete y media (Spain), Sette e mezzo (Italy), Siete y medio (Latin America).

History

Seven and a Half is a gambling game that is popular in Italy (as Sette e Mezzo), in Spain (as Siete y Media) in Brazil (as Sete e Meio) and probably in other places as well. In Italy it is traditionally played around Christmas. It is a popular game with deep roots and tradition. The objective is to collect cards that total as near as possible to 7½ without exceeding that number.

Its origins are totally uncertain but it is supposed to be based on bets that were to guess the card would take to cut the deck. It probably dates back to the 18th century.

Right above you see a Spanish deck printed in Valencia, in 1778. Below: Spanish deck made by Phelippe Ayet, Century XVI. Model of 1574 found in the house and tower of Lujanes. Museo español de antigüedades. 1874, tomo 3.

References and Links

https://en.wikipedia.org/wiki/Sette_e_mezzo

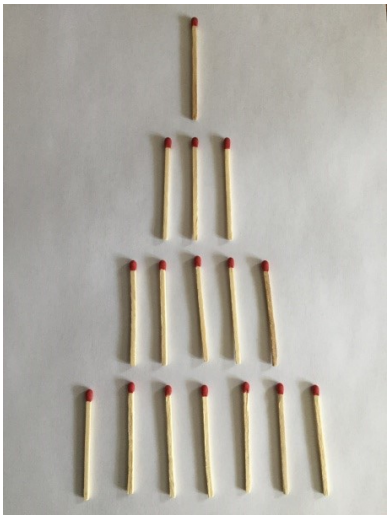
https://es.wikipedia.org/wiki/Siete_y_media

http://www.ludoteka.com/seven-and-a-half.html

https://es.wikipedia.org/wiki/Baraja_espa%C3%B1ola

YouTube Math-GAMES-Channel: https://www.youtube.com/watch?v=d7WnfJqe07s

9.3 Nim Game



Nim is an ancient game of obscure origin, in which two players alternate in removing objects from different piles, with the player who removes the last object winning in the normal play variant and losing in another common variant.

Overview

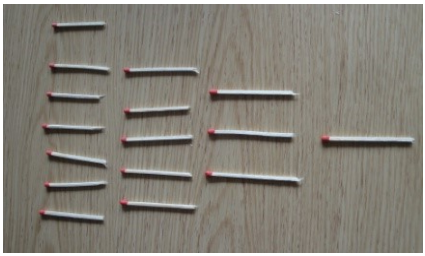
- **Genre:** mathematical game of strategy
- **Players:** 2
- **Age range:** older than 7
- **Setup Time:** less than 1 minute
- **Playing Time:** 1-2 minutes
- **Random Chance:** none

Game rules

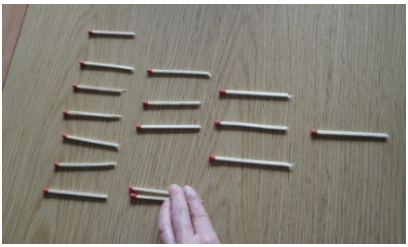
Nim is a mathematical game of strategy in which two players take turns removing objects from distinct heaps. On each turn, a player must remove at least one object, and may remove any number of objects provided they all come from the same heap. The goal of the game is not to be the player to remove the last object.

It's clear that there is no luck involved here. You can work out the best move to make by cleverly predicting the sequence of moves that would follow it.

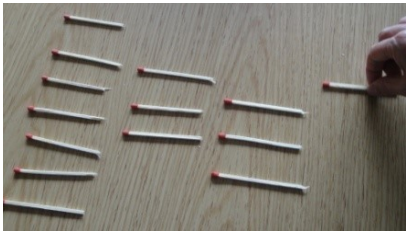
We put the matches in four rows with 1, 3, 5 and 7 elements.



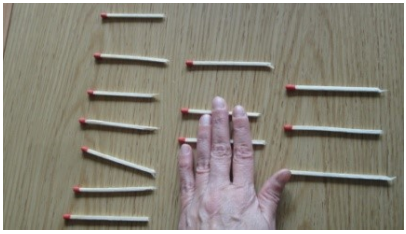
The first player removes a number of matches from a row.



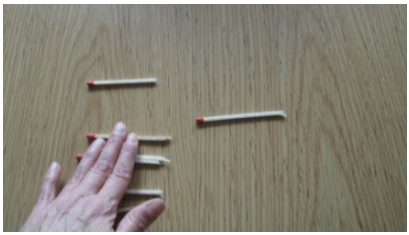
The other player takes one or more matches from the same or another row.



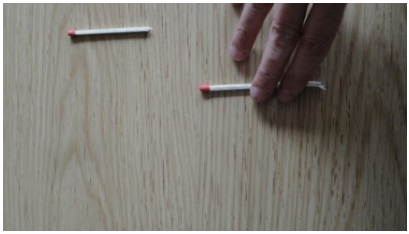
The first one plays again removing some matches from another row…



… and so on …



…until there is only one match left, so the next player is losing the game.



Nim is usually played as a *misère* game, in which the player to take the last object loses. But Nim can also be played as a normal play game, which means that the person who makes the last move (i.e., who takes the last object) wins. This is called normal play because most games follow this convention, even though Nim usually does not.

STRATEGY

The American mathematician Charles Bouton (1869-1922) set himself the daunting task of analysing the game completely. In 1902 he found that the strategy to win hinges on writing the number of elements in each heap in binary, and then adding those numbers up, not using the ordinary way of adding numbers, but something appropriately called *Nim addition*. Every combination of the objects may be considered "safe" or "unsafe"; if the position left by a player after his move assures a win for that player, the position is called safe. Every unsafe position can be made safe by an appropriate move, but every safe position is made unsafe by any move. To determine whether a position is safe or unsafe, the number of objects in each pile may be expressed in binary notation: if each column adds up to zero or an even number, the position is safe.

To add some given binary numbers using *Nim addition*, you first write them underneath each other, as you might for ordinary addition. Then you look at each of the columns in turn. If the number of 1s in a column is odd, you write a 1 underneath it; if it's even, you write a 0 underneath it. Doing this for each column gives a new binary number, and that's the result of the Nim addition.

When Charles Bouton analysed the game of Nim, he figured out two facts which hold the key to the winning strategy.

Fact 1: Suppose it's your turn and the Nim sum of the number of objects in the heaps is equal to 0. Then whatever you do, the Nim sum of the number of objects after your move will not be equal to 0.

Fact 2: Suppose it's your turn and the Nim sum of the number of objects in the heap is not equal to 0. Then there is a move which ensures that the Nim sum of the number of objects in the heaps after your move is equal to 0.

Now suppose you are player A, so you go first. Also suppose that the Nim sum of the number of objects in the heaps is not equal to 0. Your strategy will be this: if possible always make a move that reduces the next Nim sum, the Nim sum after your move, to 0. This would then mean that whatever player B does next, by fact 1 the move would turn the next Nim sum into a number that's not 0.

This shows that if the Nim sum of objects in the heaps at the start of the game is not 0, then player A has a winning strategy. The strategy is to always make a move that reduces the next Nim sum to 0.

If the Nim sum of objects in the heaps at the start of the game is equal to 0, then player B has a winning strategy. Whatever player A does on the first move will result in a non-zero Nim sum when it's B's turn. And by the same reasoning as above, this means that the winning strategy is now in B's hands.

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 10 items
- Add and subtract single-digit numbers with totals to 10

Data and statistical measures

- Understand simple information from the order of the matches at the beginning of the game (distribution per row)

Probability

- Understand probability in order to decide the number of matches to remove
- Identify the possibilities of success each time we remove a match

History

Variants of Nim have been played since ancient times. The game is said to have originated in China, it closely resembles the Chinese game of *Tsyan-shizi* or "picking stones", but the origin is uncertain.

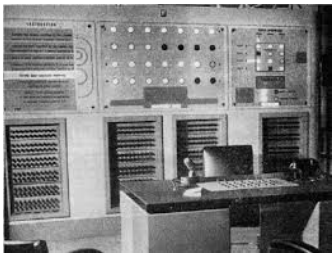
Games of this sort seem to be widely played the world over. The game of pebbles, also known as the game of odds, is played by two people who start with an odd number of pebbles placed in a pile. Taking turns, each player draws one, or two, or three pebbles from the pile. When all the pebbles have been drawn the player who has an odd number of them in his possession wins.

Predecessors of these games, in which players distribute pebbles, seeds, or other counters into rows or holes under varying rules, have been played for centuries in Africa and Asia, where they are called *mancala*.

The earliest European references to Nim are from the beginning of the 16th century. Its current name was coined by Charles L. Bouton of Harvard University, who also developed the complete theory of the game in 1901, but the origins of the name were never fully explained. The name is probably derived from German *nimm* meaning *take* (imperative), or the obsolete English verb *nim* of the same meaning.

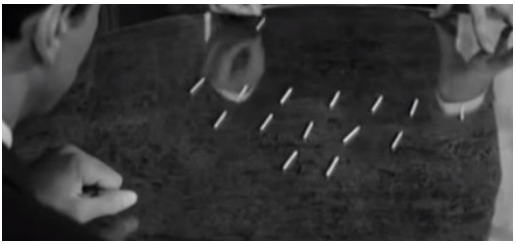
One of the first ever gaming computers, called *Nimrod*, was designed to play the game of Nim and exhibited at the 1951 Festival of Britain. The *Nimrod*, built in Britain by Ferranti for the 1951 Festival of Britain, was an early computer custom-built to play a computer game, one of the first games developed in the early history of video games. Using a panel of lights for its display, it was designed exclusively to play the game of Nim; moves were made by players pressing buttons which corresponded with the lights. *Nimrod* could play either the traditional or "reverse" form of the game.

The machine was twelve feet wide, nine feet deep, and five feet tall. It was based on an earlier Nim-playing machine, *Nimatron*, designed by Edward Condon and built by Westinghouse Electric in 1940 for display at the New York World's Fair. *Nimatron* had been constructed from electromechanical relays and weighed over a ton.



The Nimrod was intended to demonstrate Ferranti's computer design and programming skills rather than to entertain, though Festival attendees were more interested in playing the game than the logic behind it. After its initial exhibition in May, the Nimrod was shown for three weeks in October 1951 at the Berlin Industrial Show before being dismantled (picture of the Nimrod exhibit at the Computerspielemuseum in Berlin).

A version of Nim is played in the French New Wave film *Last Year at Marienbad*, a 1961 French film directed by Alain Resnais.



Last Year at Marienbad is famous for its enigmatic narrative structure, in which truth and fiction are difficult to distinguish, and the temporal and spatial relationship of the events is open to question. It turns out that M (whose behaviour seems quite irrational) proposes different characters, including the mysterious X, play some games that surprisingly always wins. One of them is Nim. Such popularity gave the film to this game that in many places is known even today precisely as *Marienbad*. The game has a default beforehand winner, the second in play (M looks very courteous start leaving his opponent but nothing complimentary). In other words, if the second player uses the optimal strategy, he will win whatever the first player does.

References and Links

Wikipedia: https://en.wikipedia.org/wiki/Nim - https://es.wikipedia.org/wiki/Nim_(juego)

https://en.wikipedia.org/wiki/Early_history_of_video_games

Other Links: https://matemelga.wordpress.com/2013/09/01/el-juego-del-nim/

http://www.librosmaravillosos.com/matematicaparadivertirse/seccion06.html

https://plus.maths.org/content/play-win-nim - http://www.britannica.com/topic/nim

10.1 Okey (Rummikub)

Overview

- **Genre:** tile based game, similar to the card game rummy
- **Players:** 2 or 4
- **Age range:** older than 12
- **Setup Time:** less than 3 minutes
- **Playing Time:** 30 minutes to 3 hours
- **Random Chance:** no random, but you use dice for selecting

General

"Okey" is a tile-based game, very popular in Turkey. It is almost always played with 4 players, though in principle can be played with two or three. It is very similar to the game Rummikub as it is played with the same set of boards and tiles but with different rules. The game apparently evolved from the original Rummikub through cultural contacts of Gastarbeiter in Germany. In Turkey and among Turkish communities abroad, it is very popular not only at homes but also at coffeehouses. The game was invented 1930 by E. Hertzano and combines elements of the card game Rummy and the tile game Mahjong.

Game rules

1st Phase

The first dealer is chosen at random. After the hands have been played and scored, the turn to deal passes to the right.

The 106 tiles are placed face down on the table and thoroughly mixed. Then the players set them up into 21 stacks of five tiles, the tiles in each pile being face down. One tile is left over - this is temporarily kept by the dealer.

There is no specific rule about how many stacks should be in front of each player. It is convenient to have at least six in front of the dealer, but this makes no real difference to the game.

The dealer now throws the dice twice. The result of the first throw selects one of the tile stacks in front of the dealer, counting from left to right. The dealer places the single remaining tile on top of this selected stack. If the number thrown is greater than the number of stacks in front of the dealer, then the count will continue using the stacks in front of the player to dealer's right, and one of these will be selected. The selected stack now has six tiles.

The second throw of the dice selects one of the tiles in the selected stack, counting upwards from the bottom of the stack. The selected tile is extracted from the stack and placed face up on top of it. If the selected tile is a false joker, it is returned to the selected stack and the second throw of the die is repeated until a numbered tile is selected.

2ND PHASE

This face up tile determines the "joker" (okey) for the game - a wild tile that can be used to represent other tiles to complete a combination. The joker is the tile of the same colour and one number greater than the face up tile. For example if the face up tile is the green 10, the green 11s are jokers. The false jokers are not wild - they are used only to represent the tiles that have become jokers. So for example when the green 11s are jokers, the false jokers are played as green 11s (and cannot represent any other tile). If the face up tile is a 13, the 1s of the same colour are jokers.

Now the stacks of tiles are distributed to the players. The player to dealer's right will receive 15 tiles and the others 14 each. The player to the right of the dealer takes the next stack after (to the right of) the selected stack with the face up tile

on top of it, then the player opposite the dealer takes the following stack, and so on anticlockwise around the table, until each player has two stacks (10 tiles). Now the player to the dealer's right receives the whole of the next stack, but the player sitting opposite the dealer is given only the top 4 tiles of the following stack. The player to the dealer's left receives the last tile of this stack and 3 tiles from the top of the next stack, and finally the dealer takes the last 2 tiles from this stack and 2 from the next stack.

All the players should arrange their tiles so that they can see their faces but the other players cannot. Wooden racks are often used for this. The remaining tiles are left for the players to draw from during the game. They are moved to the middle of the table, without looking at them or disturbing their order.

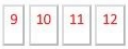
PLAY

Before the play begins, if any player holds the tile that matches the face up tile on top of the last stack of six tiles, the player may show that tile, and score one point.

Now the player to the dealer's right begins the play by discarding one tile, face up. After this, each player in turn may either take the tile just discarded by the previous player, or draw the next tile from the supply in the centre of the table, and must then discard one unwanted tile. This continues in anticlockwise rotation until a player forms a winning hand and exposes it, ending the play.

Discarded tiles are placed to the right of the player who discarded them, in a stack, so that only the most recent discard in the stack is visible.

The usual rule is that you are allowed to look through all the tiles in the discard stacks to your right (the tiles you discarded) and to your left (the tiles you had an opportunity to take), but you can only see the exposed top tiles of the two discard stacks on the other side of the table.

The objective of the game is to collect sets and runs. A run is a row of following numbers in the same colour 

and a set is a group of minimum 3 same numbers in different colours 

.

A set consists of three tiles (üçlü) or four tiles (dörtlü) of the same number and different colours. So for example a black 7 plus two red 7s would not form a valid set.) A run (el) consists of three or more consecutive tiles of the same colour. The 1 can be used as the lowest tile, below the 2, or as the highest tile, above the 13,

but not both at once. So green 1-2-3 or yellow 12-13-1 would be valid runs, but black 13-1-2 would not be valid. A winning hand consists of 14 tiles formed entirely into sets and runs - for example two sets of 3 and two runs of 4, or a run of 6 plus a run of 3 plus a set of 4. No tile can be used as part of more than one combination (set or run) at the same time.

If you have a winning hand, then you can end the play by exposing all 14 of your tiles after discarding. Apart from the discards and the face up tile on top of the six-tile stack, no tiles are exposed until a player shows a winning hand: no sets or runs are exposed during the game.

Tiles are always drawn from the top of the next available stack. When only the final stack of 6 tiles remains, the exposed tile is removed from the top of this stack and the other five tiles are drawn in order.

As already explained, the two tiles that are the same colour as the face up tile and one greater in number are the jokers. These tiles can be used to represent any tile the holder desires, in order to complete a set or run. For example, if the red 4 is face up, the red 5s are jokers. {green 6, red 5, red 5, green 9} would count as a run, using jokers for the green 7 and 8. Alternatively, {yellow 10, black 10, red 5} would be a set, using the red 5 to represent the red or green 10.

The two false jokers - the tiles without numbers - are used only to represent the joker tiles. So for example when red 5s are jokers, the false jokers are played as red 5s: for example {red 4, false joker and red 6} is a run, and {black 5, green 5, yellow 5, false joker} is a set.

Each group must have same numbers in different colours: 7 7 7

Each group must be same colour with ordered numbers:

6 7 8 9 or 10 11 12 1

More information and a game example

https://www.youtube.com/watch?v=BdqXv0qZJ9I

https://www.youtube.com/watch?v=FKjw2OQ1aaQ

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 9 items
- Read and write numbers up to 9, including zero
- Order and compare numbers up to 10, including zero
- Add single-digit numbers with totals to 10
- Subtract single-digit numbers from numbers up to 10
- Draw a number line and sort numbers
- Understand the coordinate system

History

Okey game is based on one of the Asian historic game Domino from the 13th century. First Okey was played in China and then Persian played Domino game. Arabs also played the game and adapted wooden boards to play easily. Arabs called it “El Turaft”, this means “in ordered numbers”. Turks met with this game in 15th century by relations with Arabs.

References and Links

Wikipedia: https://de.wikipedia.org/wiki/Okey_(Spiel)

https://en.wikipedia.org/wiki/Okey

https://tr.wikipedia.org/wiki/Okey

Computer simulated games:

http://de.download.cnet.com/Okey/3000-18516_4-10920175.html

http://www.rummyroyal.com/okey/download-software.html

10.2 Hopscotch (Seksek)

- GENRE: outdoor games, urban or rural environment
- PLAYERS: 2 to 4 or more
- AGE RANGE: 5 years and up
- SETUP TIME: 1 minute
- PLAYING TIME: about 30 minutes
- RANDOM CHANCE: yes, but minimal

Hopscotch is a children's game that can be played with several players or alone. Hopscotch is a popular playground game in which players toss a small object into numbered spaces of a pattern of rectangles outlined on the ground and then hop or jump through the spaces to retrieve the object

Game Rules

Stage 1: The playground

The playground is the street, the pavement, the park, the playing ground, the school yard. Squares and two semicircles are drawn on the ground, numbered from 1 to 10 (sometimes from 1 to 8). We draw with a piece of chalk, brick or paint.

Stage 2: Throwing the pebble

Each player has a pebble. The players throw them successively when their turn comes. If a pebble lands on the line, the respective player can repeat the throw. If he fails to aim the square, some other player's turn comes.

Stage 3: Jumps

The players jump from 1 to 10 and backwards as follows: they jump on one leg in the boxes with simple numbers and on both legs on adjoining boxes. When they return, they bend to pick up the pebble. If they tread on the line or outside the boxes they are penalized and repeat the throw. The players 'vocabulary includes: "You trod the line! Repeat"; "You forgot to pick up the pebble! Repeat!"

All the players have fun; they vividly communicate and leave the drawing on the pavement as a sign of an unforgettable contest.

Game Description

To play hopscotch, a court is first laid out on the ground. Depending on the available surface, the court is either scratched out in dirt, or drawn with chalk on pavement. Courts may be permanently marked where playgrounds are commonly paved, as in elementary schools. Designs vary, but the court is usually composed of a series of linear squares interspersed with blocks of two lateral squares. Traditionally the court ends with a "safe" or "home" base in which the player may turn before completing the reverse trip. The home base may be a square, a rectangle, or a semicircle. The squares are then numbered in the sequence in which they are to be hopped.

The first player tosses the marker (typically a stone, coin or bean bag) into the first square. The marker must land completely within the designated square and without touching a line or bouncing out. The player then hops through the course, skipping the square with the marker in it. Single squares must be hopped on one foot. For the first single square, either foot may be used. Side by side squares are straddled, with the left foot landing in the left square, and the right foot landing in the right square. Optional squares marked "Safe", "Home", or "Rest" are neutral squares, and may be hopped through in any manner without penalty. After hopping into the "Safe", "Home" or "Rest" the player must then turn around and return through the course (square 9, then squares 8 and 7, next square 6 and so forth) on one or two legs depending on the square until he or she reaches the

square with their marker. They then must retrieve their marker and continue the course as stated without touching a line or stepping into a square with another player's marker.

Upon successfully completing the sequence, the player continues the turn by tossing the marker into square number two, and repeating the pattern.

If, while hopping through the court in either direction, the player steps on a line, misses a square, or loses balance, the turn ends. Players begin their turns where they last left off. The first player to complete one course for every numbered square on the court wins the game.

Although the marker is most often picked up during the game, historically, in the boy's game, the marker was kicked sequentially back through the course on the return trip and then kicked out.

History

A hopscotch game with a traditional magpie rhyme in Morecambe, England. It is attested that an ancient form of hopscotch was played by Roman children, but the first recorded references to the game in English-speaking world date back to the late 17th century, usually under the name "scotch-hop" or "scotch-hopper(s)".

A manuscript Book of Games compiled between 1635 and 1672 by Francis Willoughby refers to 'Scotch Hopper. They play with a piece of tile or a little flat piece of lead, upon a boarded floor, or any area divided into oblong figures like boards'.

Since the game was known and popular in the seventeenth century, it is logical to suppose it may have existed at least a few decades (or perhaps even many centuries) before its earliest literary reference; but no conclusive evidence has yet been presented to support this theory.

Hop-scotch is largely known in Europe, in France, Bulgaria and Turkey. We call it “sotron” in Romanian, the French call it “marelle”, the English “hop-scotch”, the Italians “campana”, the Spaniards “’rayuela”, etc. The linguist Rodica Zafiu searches the origin of the word in Romanian.

The Name

According to the Oxford English Dictionary the etymology of hopscotch is a formation from the words "hop" and "scotch", the latter in the sense of "an incised line or scratch". The journal of the British Archaeological Association, Volume 26 (dated March 9, 1870) states, "The sport of Hop-Scotch or Scotch-Hoppers" is called in Yorkshire "Hop-Score", and in Suffolk "Scotch Hobbies" from the boy who gets on the player's back whilst hopping, as it is there termed.

Synonyms and Variants

There is a variant in which the player is not allowed to touch the pebble with his/her hand, but it is slightly pushed by the leg decreasingly down to 1.

In another variant the digits 9 and 10 are replaced by the words "sky" and "earth".

A variant of my childhood was called "the Man" according to the resemblance between the drawing and a stylized man. It was only later on that I found out that the game was also called şotron.

On the right you see a variation in the entrance of an official building in Belo Horizonte, Brazil and a street mosaic in the shape of hopscotch game in Boston.

There are many other forms of hopscotch played across the globe. In India it is called Stapu or Kith-Kith, in Spain and some Latin American countries, it's rayuela, although it may also be known as golosa. In Turkey, it is Seksek (the verb "sek" means to jump with one foot). In Russian it is known as классики (diminutive for the word meaning classes).

Calculator Hopscotch

See chapter 6.3!

Peevers or Peever

In the Glasgow area, the game is called "beds" or "Peever(s)". "Peever" is also the name of the object which is slid across the grid to land in a square. In the 1950s and 1960s in Glasgow, it was common for the peever to be a shoe polish tin filled with stones or dirt and screwed shut.

Escargot

A French variant of hopscotch is known as escargot (snail) or ronde (round hopscotch). It is played on a spiral course. Players must hop on one foot to the centre of the spiral and back out again. If the player reaches the centre without stepping on a line or losing balance he or she marks one square with his or her initials, and from then on may place two feet in that square, while all other players must hop over it. The game ends when all squares are marked or no one can reach the centre, and the winner is the player who "owns" the most squares.

Himmel und Hölle

In Germany, Austria, and Switzerland the game is called “Himmel und Hölle” (Heaven and Hell), although there are also some other names used, depending on the region. The square below 1 or the 1 itself are called Erde (Earth), while the second to last square is the Hölle (Hell) and the last one is Himmel (Heaven). The first player throws a small stone into the first square and then jumps to the square and must kick the stone to the next square and so on, however, the stone or the player cannot stop in Hell so they try to skip that square.

What Mathematical Content can be Learned?

Numeracy

- Count reliably up to 10 items
- Add and subtract single digit numbers

Geometry

- Drawing 2-dimensional figures: square, rectangle, circle, semicircle, lines
- Recognize and name two dimensional shapes
- Describe length and width of shapes

Resources and Links

Wikipedia: https://en.wikipedia.org/wiki/Hopscotch

Other Links: http://www.infatablocului.ro/ - http://www.romlit.ro/otron

YouTube: https://www.youtube.com/watch?v=w645hRDKUu4

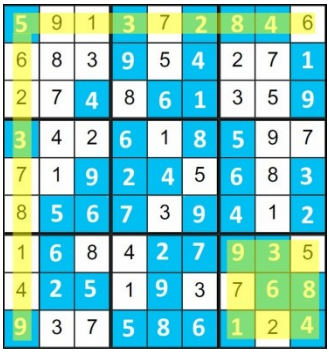
https://www.youtube.com/watch?v=qIp27jTp4TU

10.3 SUDOKU

OVERVIEW

- **Genre:** logical math game, puzzle type
- **Players:** 1 player
- **Age group:** 7 +
- **Duration:** indefinite
- **Choice:** random based on card shuffler

Sudoku is one of the most famous mathematical puzzles in the world. It consists of 81 cells in which a fixed number of digits from 1 to 9 are placed. The task is to fill the remaining empty cells considering the following rules: In the game participate only **the numbers from 1 to 9**. The game board (square 9x9) must be filled in such pattern that each digit from 1 to 9 to be present in each **column**, in each **row**, and in each **small square** (3x3) only once. On the right side above you see the beginning of the game, where you should fill in the numbers. Below is the solution – try it, but don't look!



GAME RULES

The Sudoku rules are very easy, but the puzzle can be either easy or very complicated, depending on the given Sudoku numbers.

Sudoku is a logic-based, combinatorial number-placement puzzle. The objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids that compose the grid (also called "boxes", "blocks", "regions", or

"sub-squares") contains all of the digits from 1 to 9. The puzzle setter provides a partially completed grid, which for a well-posed puzzle has a unique solution.

More information about the game

https://www.youtube.com/watch?v=uVrM8q4pY44&list=PLAhxvOuSHpkYsUufBeuHjQUNcb3iHt9nl&index=2

https://www.youtube.com/watch?v=cF5cMCjFzSg

What Mathematical Content can be Learned?

Numeracy

- Read and write numbers up to 9
- Order and compare numbers up to 9

Geometry

- Recognize and name two dimensional shapes (square, rectangle)
- Measure areas by counting squares or using grids

History

Number puzzles appeared in newspapers in the late 19th century, when French puzzle setters began experimenting with removing numbers from magic squares. Le Siècle, a Paris daily, published a partially completed 9×9 magic square with 3×3 sub-squares on November 19, 1892.

The modern Sudoku was most likely designed anonymously by Howard Garns, a 74-year-old retired architect and freelance puzzle constructor from Connersville, Indiana, and first published in 1979 by Dell Magazines as Number Place (the earliest known examples of modern Sudoku). Garns's name was always present on the list of contributors in issues of Dell Pencil Puzzles and Word Games that included Number Place, and was always absent from issues that did not. He died in 1989 before getting a chance to see his creation as a worldwide phenomenon.

The puzzle was introduced in Japan by Nikoli in the paper Monthly Nikolist in April 1984 as Sūji wa dokushin ni, which also can be translated as "the digits must be single" or "the digits are limited to one occurrence." Sudoku is a registered

trademark in Japan and the puzzle is generally referred to as Number Place. The Times of London began featuring Sudoku in late 2004 after a successful appearance in a local US newspaper, from the efforts of Wayne Gould, and rapidly spread to other newspapers as a regular feature. Gould devised a computer program to produce unique puzzles rapidly.

Variants

Variations of grid sizes

Although the 9×9 grid with 3×3 regions is by far the most common, many other variations exist. Sample puzzles can be 4×4 grids with 2×2 regions; 5×5 grids with pantomime regions have been published under the name Logi-5; the World Puzzle Championship has featured a 6×6 grid with 2×3 regions and a 7×7 grid with six heptomino regions and a disjoint region. Larger grids are also possible. The Times offers a 12×12-grid "Dodeka Sudoku" with 12 regions of 4×3 squares.

Mini Sudoku

A variant named "Mini Sudoku" appears in the American newspaper USA Today and elsewhere, which is played on a 6×6 grid with 3×2 regions. The object is the same as that of standard Sudoku, but the puzzle only uses the numbers 1 through 6. A similar form, for younger solvers of puzzles, called "The Junior Sudoku", has appeared in some newspapers, such as some editions of The Daily Mail.

References and Links

Wikipedia:

https://en.wikipedia.org/wiki/Sudoku

More Links and Online Games:

http://sudoku.soeinding.de/sudokuAusdrucken.php

http://www.websudoku.com/

http://www.sudoku.com/?lang=de

SUDOKU

NUMBER PUZZLE

Task: Fill the remaining empty cells considering the following rules: The game board (square 9x9) must be filled in such pattern that each digit from 1 to 9 to be present in each column, in each row, and in each small square (3x3) only once.

	2		9		8	4	3	
			3		7		2	6
5	3			2		1		
	9		6			5	7	
3	7	4				2		8
8					4		9	
		5		3	2			
4	1	2			9	8		3
		3			5		1	2

	5	1		4	2	7		
		2			8	5		
9	6				3	8	1	2
6				7		3	8	9
		9						
	8				6		4	
4		8			7	9		3
	2	6		3	9	4		
7	9					2	5	1

4	8	6	1	3		5		7
		2				6		8
	6				5	1		4
	5		6	4			8	
2		9	3	1		7	5	6
5				6	1		9	2
	2		4				7	5
	3	8		5	2			

8			5	2	7	9		6
9	1						2	7
			4				3	5
2		7	1		4	3	5	9
4				5		6		
	5	3	7	6	9			
								3
3	4	8		7			6	
	9		6				8	4

SUDOKU

NUMBER PUZZLE

Solution:

6	2	7	9	1	8	4	3	5
1	4	8	3	5	7	9	2	6
5	3	9	4	2	6	1	8	7
2	9	1	6	8	3	5	7	4
3	7	4	5	9	1	2	6	8
8	5	6	2	7	4	3	9	1
7	8	5	1	3	2	6	4	9
4	1	2	7	6	9	8	5	3
9	6	3	8	4	5	7	1	2

8	5	1	9	4	2	7	3	6
3	7	2	6	1	8	5	9	4
9	6	4	7	5	3	8	1	2
6	4	5	2	7	1	3	8	9
1	3	9	4	8	5	6	2	7
2	8	7	3	9	6	1	4	5
4	1	8	5	2	7	9	6	3
5	2	6	1	3	9	4	7	8
7	9	3	8	6	4	2	5	1

4	8	6	1	3	9	5	2	7
3	9	2	5	7	4	6	1	8
7	1	5	2	8	6	9	4	3
8	6	7	9	2	5	1	3	4
1	5	3	6	4	7	2	8	9
2	4	9	3	1	8	7	5	6
5	7	4	8	6	1	3	9	2
6	2	1	4	9	3	8	7	5
9	3	8	7	5	2	4	6	1

8	3	4	5	2	7	9	1	6
9	1	5	8	3	6	4	2	7
7	2	6	4	9	1	8	3	5
2	6	7	1	8	4	3	5	9
4	8	9	3	5	2	6	7	1
1	5	3	7	6	9	2	4	8
6	7	1	2	4	8	5	9	3
3	4	8	9	7	5	1	6	2
5	9	2	6	1	3	7	8	4

EPILOGUE

DIGITAL AMNESIA AND MATHEMATICAL GAMES

by Georgeta Adam

New technologies affect long-term memory

Have you recently tested yourself to see how many phone numbers you remember by heart? You will realize that they are very few, because you have now at hand the phone contacts always available in your mobile! Our brain no longer remembers them, because it no longer practises this kind of memory exercise! Soon, children, accustomed to tablets since early childhood, will no longer know how to multiply, divide, add, or subtract. Nowadays, we receive so much information without any effort whatsoever, that we make no longer the effort to memorize it and thus our capacity diminishes since we stimulate it no longer.

The psychologist Keren Rosner pointed out this "laziness" of our brain which takes over everything that is readily available on our smart phones. But the advent of degenerative diseases at a very early age is real, because there are several factors that seem to help us turn ourselves into robots. Recent research conducted by Kaspersky Lab in Russia has found that new technologies influence long-term memory. The research was carried out on 6.000 subjects and has shown that smart phones accelerate digital amnesia. A few years ago I went to Vilnius, Lithuania and most people were finding their way quickly by using their smart phones. The above mentioned research shows that 40 % of people are looking for information on Google and that greater trust is placed on the information online than in their own memory. This digital convenience leads to 25 % of people forgetting the information found on the Internet after use. Therefore, rapid access to information has devastating effects on long-term memory! What is going on with our memory, with all our knowledge, with our ability to make connections? The psychologist Keren Rosner brings forward arguments supported by the already mentioned study: "People no longer have the data memory they used to, before this explosion of new technologies. These technologies have nevertheless a bright side, since they help us enormously, but they also have a negative effect. In the short term, we have access to information; we can easily make correlations in a timely manner which corresponds to everyday life and to the need to meet the daily competition at work and in social life. On the other hand, memory becomes lazy, because we no longer make much use of it, we assimilate new information with difficulty and it has become ever more difficult to make new connections on our own, without the support of some new technologies".

The computer is but an extension of the brain...

A piece of information broadcast by the BBC presents University of Birmingham, UK Professor Maria Wimber's opinion, confirming the idea that

the tendency to look for information on the Internet "prevents the formation of long term memories". Professor Wimber refers to the study of the Karpesky Lab company, also quoted by us, which examined the memorizing habits of 600 000 adults in UK, France, Germany, Italy, Spain, Belgium, The Netherlands and Luxembourg, and which concluded: "Our brain seems to reinforce memory, each time we remember something, and at the same time to forget the irrelevant information that distracts us". Anyway, we have to keep in mind the idea that nowadays people use computers as an 'extension' of their brain. This has an effect of 'digital amnesia'. Convenience makes it possible for us to forget important information, convinced that we shall be able to recover it by means of a 'digital gadget'. (A study carried out in the UK on the topic of memorising telephone numbers has shown that people rely so much on the mobile phone that 71 % did not recall the telephone numbers of their children, 87% the number of schools where their children go to, 57 % the phone number at their work, 49 % the telephone number of their spouse, 47 % the phone number of their landline they had at home when they were children.)

The psychologist Matthew Fisher from Yale University, asked by the British newspaper Daily Mail, warned that this serious problem of laziness damages our mind. We sometimes think that we are smarter because Google offers us many achievements of scientists and we somehow feel these achievements are ours. "The Internet is a medium capable of offering an answer to almost any question; it's as if we have instant access to the entire science of mankind. Therefore, it is very easy to mistake one's own knowledge for the information available from an external source. When people are by their own they often realize how insecure they are about what they know and about how much they rely on the Internet."

I shall tell you a funny story about a friend of mine, I.A. Last summer she left in order to meet someone coming from Australia to quite a faraway city in Romania. She forgot her mobile phone at home, and there was no one there whom she could call. A real adventure started for the recovery of the telephone number of the person coming from Australia. She could not find the lost phone number in the e-mail she had received from the Australian visitor, because she considered the message unimportant and she deleted it. She called her husband, who was at work, but he could not help much. When she got back home, she tried to look for clues to the "lost phone number" but she did not manage to find it. Only after calling her mobile operator customer service did she manage to find the number she had dialled in order to talk to the Australian visitor. Another friend of mine, M.H. recently confessed that she knows the landline numbers she used before mobiles arrived on the market by heart, but that she could not remember any recent phone number, except at most the ones in her office.

A new innovative teaching method: Gamification

A press release highlights that a young Romanian mathematician has been designated teacher of the year in the Netherlands (2015) by a jury made of students and education practitioners in the Netherlands. They appreciated the innovative teaching method used by professor Alexandru Iosup "who uses a method based on games during his classes in order to explain complex concepts of engineering and computing to his students from the faculty of Engineering, Mathematics and Computing of the Technical University in Delft". This is the testimony made by the young teacher concerning the importance of teaching techniques to encourage innovation and inventiveness: "However, what is essentially

different from the traditional way of teaching is the method used to teach, gamification, i.e. using techniques initially developed for online games and social games. To put it differently, the course becomes a game and students have to learn in order to win". Alexandru Iosup (34 years old, graduate of the Polytechnic University of Bucharest), shows that gamification, however, is not in fact a new method, since it was used by companies 50 years ago. He added: "However, education at university level has only just begun to use «gamification»"...

Math games put the mind to work!

Can we therefore talk about the efficiency of math games in enabling our memory to store new information? Are they a means to exercise the human brain connections? The same way games are used to teach arithmetic to small pupils or even at pre-school age, math games for adult age do not mean only killing spare time or socializing. Several crosswords, Sudoku magazines and a real industry of games translated from all languages flooded the Romanian market too. The successful series Game of Thrones created characters and stories in the games industry.

Nostalgic "coffee and tea houses" have started to open, where traditional games — rummy, card games, mill — replace computer games. Parents, who were children 30 years ago, reinvented the games of their childhood, which they used to play in front of their "block", to fill their free time when there were no computers. They started to edit them and play them with their own children in order to help them give up the tablet, the smart phone, the computer in favour of open air games. Who has not played hopscotch, the skipping rope, "sausage casings entangled" or the line? Inventiveness, creativity, social networking through play in open air, in parks or on specially designed areas are elements which enhance the beauty of childhood. Lonely children who make friends among the virtual characters are affected not only as far as long-term memory is concerned, but also regarding their harmonious physical development. Outdoor games which use several math elements (arithmetic, geometry) have also a sporting dimension that increases their value.

"One sheep, two sheep, three sheep. Following the wool..."

This is a funny phrasing in a book written by David Berlinski: One, two, three - Absolutely elementary Maths (Paris, 2011). The French author convinces us that the world of figures is mysterious and that modern maths only discovers the "archaic maths" hidden in the world order, nature and the universe. Another author, Ian Stewart, asks us some questions in his book the Nature's Numbers (The Unreal Reality of Mathematical Imagination) based on some funny mathematical mysteries: why are there flowers with five or with 8 petals, but so very few with six or seven? Why do snowflakes have a hexagonal symmetry? Why do tigers have stripes and leopards spots? The natural world seen through the eyes of the mathematicians is based on the rationale that places man in the universe. Math games have led to different applications, including the discoveries made in the world of computers. Mathematics is arid, difficult, and weird for many, but we cannot live without them, which is odd. Berlinski himself says:

"The laws of mathematics. This is a strange formula, the more so as the mathematicians who discovered them were lawyers."

Let us have fun, by counting every day with ghost numbers which are part of the existence of human achievements, including the internet and Facebook ... In other words, back to math games to keep our memory fresh and alive.

Some ideas to be noted

New technology affects long-term memory - Kaspersky Lab research, Russia.
Modern technologies lead to addiction (Keren Rosner, psychologist).
Scientists warn that Google influences a generation of people who are not as smart as they sometimes think they are: sometimes when they have to manage on their own, they find themselves at a loss (psychologist Matthew Fisher, Yale University).
Recommendations for putting the memory to work:

- Keren Rosner: "The memory should always be put to work, since this is the only way in which it remains active for a long time and it protects us from degenerative diseases and cognitive problems which arise with age. Persons, who do not put their memory to work for a long period of time will be confronted with such problems sooner or later and will not enjoy a clear mind into old age...
- Simply reading the information on the internet is useful only in the short term, but in order to be able to treasure it for a long time it should be repeated either in writing or orally. Each person has a specific type of memory storage: some have visual memory, they treasure information by looking, and others need to repeat or to write in order to be able to remember information".
- Learning a foreign language, which proved to be helpful for the memory irrespective of age, but it is also important to have as many fields of interest as possible.
- Memorising telephone numbers is important as it is a string of numbers that we reactivate regularly.
- Favourite quotations that we can use in discussions.
- Poetry lines.
- Our recommendation: games, math games.

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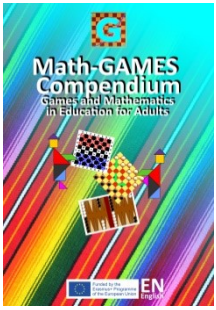

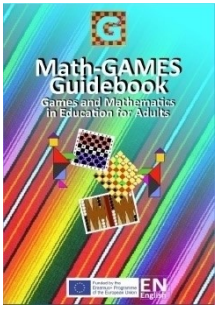

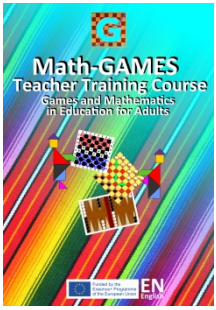

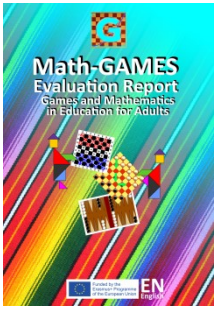

AVAILABLE MATERIAL IN THE MATH-GAMES PROJECT

The four parts of the output of the project Math-GAMES:
„**Math-GAMES Compendium of Famous Traditional Games**“, which are books in nine languages (BG, DE, EN, ES (Cast. und Val.), FR, GR, IT, RO).
After that the partners of the project will prove how traditional games could be implemented in their learning program for a better understanding Mathematics, especially for lower skilled people, for young people and for immigrants, if there are needs.
The results are the “**Math-GAMES Numeracy Learning Guidebook (Mathematical Literacy)**” in nine languages (BG, DE, EN, ES (Cast. und Val.), FR, GR, IT, RO).
In the third part of the project the project partners will prove by doing and testing during real courses and seminars that playing games between people with different skills assist in social integration and thus traditional games will be saved by transferring them to other people and they will not be lost.
The result is a „**Math-GAMES Teacher Training Course and Seminar**“, which is held for the next years in different countries. The e-presentation, the seminar and the teacher training course are published in English.
Finally the „**Math-GAMES Testing and Evaluation Report**“ is published. It is a report about the project, the work, the activities during the lessons, the competitions in schools, the meetings and the evaluation. The Math-GAMES Project Report is published in English. All the Material is available from 2018.

Information:
Website: www.math-games.eu

Math-GAMES Compendium

978-3-89697-800-4

EN English